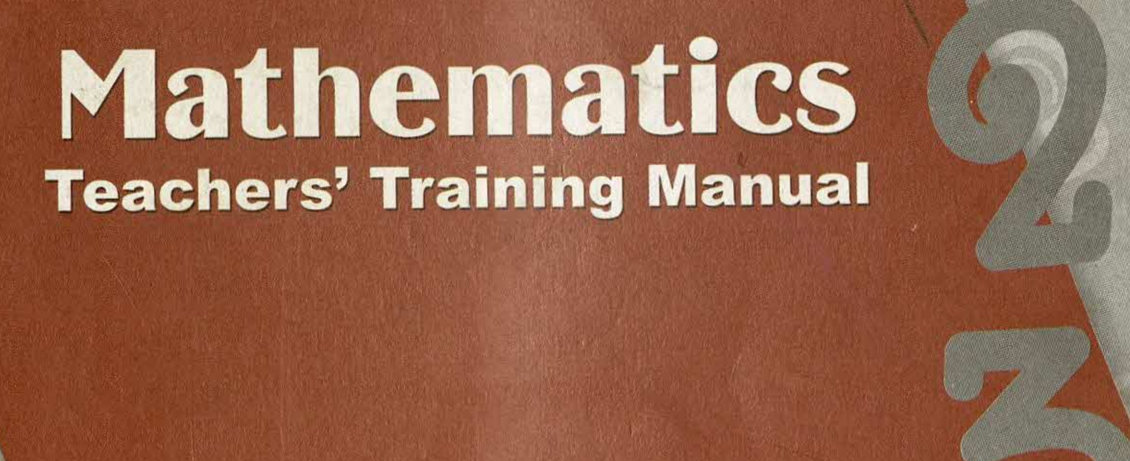
For Acc
SCERT lib
20/6/11

Mathematics

Teachers' Training Manual



For Classes 1 and 2

CHILDREN'S BILL OF RIGHTS

A child is every person under the age of 18 years. Parents have the primary responsibility for the upbringing and development of the child. The State shall respect and ensure the rights of the child.

- I have the Right to express my views freely which should be taken seriously, and everyone has the Responsibility to listen to others. *(Article 12,13)*
- I have the Right to good health care, and everyone has the Responsibility to help others get basic health care and clean water. *(Article 24)*
- I have the Right to a good education, and everyone has the Responsibility to encourage all children to go to school. *(Article 28,29,23)*
- I have the Right to be loved and protected from harm and abuse, and everyone has the Responsibility to love and care for others. *(Article 19)*
- I have the Right to be included whatever my abilities, and everyone has the Responsibility to respect others for their differences. *(Article 23)*
- I have the Right to be proud of my heritage and beliefs, and everyone has the Responsibility to respect the culture and belief of others. *(Article 29,30)*
- I have the Right to a safe and comfortable home and everyone has the Responsibility to make sure all children have homes. *(Article 27)*
- I have the Right to make mistakes, and everyone has the Responsibility to accept we can learn from our mistakes. *(Article 28)*
- I have the Right to be well fed, and everyone has the Responsibility to prevent people from starving. *(Article 24)*
- I have the Right to a clean environment, and everyone has the Responsibility not to pollute it. *(Article 29)*
- I have the Right to live without violence (verbal, physical, emotional), and everyone has the Responsibility not to be violent to others. *(Article 28,37)*
- I have the Right to be protected from economic and sexual exploitation, and everyone has the Responsibility to ensure that no child is forced to work and is given a free and secure environment. *(Article 32,34)*

These rights and responsibilities are enshrined in the United Nations Convention on the Rights of the Child, 1989. It contains all the rights which children and young people have all over the world. The Government of India signed this document in 1992.

Mathematics

Teachers' Training Manual

For Classes 1 and 2

S. C. E. R. T. West Bengal
Library
Calcutta
25/3, B. C. Road

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

First Edition
October 2010 Asvina 1932

PD 2T SU

© National Council of Educational Research and Training, 2010

04.07.2011
14563

₹ 150.00

Published at the Publication Department by the Secretary, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 110 016 and printed at Deekay Printers, 5/16, Kirti Nagar Industrial Area, New Delhi - 110 015

ISBN 978-93-5007-135-9

ALL RIGHTS RESERVED

- No part of this publication may be reproduced, stored in a retrieval system or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise without the prior permission of the publisher.
- This book is sold subject to the condition that it shall not, by way of trade, be lent, re-sold, hired out or otherwise disposed of without the publisher's consent, in any form of binding or cover other than that in which it is published.
- The correct price of this publication is the price printed on this page, Any revised price indicated by a rubber stamp or by a sticker or by any other means is incorrect and should be unacceptable.

OFFICES OF THE PUBLICATION DEPARTMENT, NCERT

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 **Phone : 011-26562708**

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bangluru 560 085 **Phone : 080-26725740**

Navjivan Trust Building
P.O.Navjivan
Ahmedabad 380 014 **Phone : 079-27541446**

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 **Phone : 033-25530454**

CWC Complex
Maligaon
Guwahati 781 021 **Phone : 0361-2674869**

Publication Team

Head, Publication Department : *Neerja Shukla*

Chief Production Officer : *Shiv Kumar*

Chief Editor : *Shveta Uppal*

Chief Business Manager : *Gautam Ganguly*

Production Assistant : *Mukesh Gaur*

Cover, Layout Design and Illustrations

Blue Fish

Foreword

This manual for mathematics teachers who work in the early primary classes attempts to address several familiar problems that children's education in India has faced for a very long time. Salient among these problems is the tendency to force children to memorise by rote and to copy numbers from the blackboard. The conventional pedagogy of mathematics treats it as an abstract subject which only a few can master. This kind of approach has no relevance or meaning in the context of universalisation of elementary education. Now that every Indian child has a fundamental right to elementary education, the application of child-centred methods of teaching should be our highest priority. This manual offers a range of elaborate strategies for the use of such methods in Classes I and II. The focus is on using mathematics for promoting logical thinking and reasoning by selecting and designing activities related to children's everyday life. The manual emphasises the importance of conceptual clarity and skill-development in teachers. It addresses issues like the use of simple language in communicating mathematical ideas and encouraging children to articulate the strategies they themselves develop for solving mathematical problems. Systemic issues like the use of textbooks and resources available in the local environment have also been discussed.

This document has been developed by NCERT as part of a special programme in early mathematics supported by *Sarva Shiksha Abhiyan* (SSA). The Department of Education in Science and Mathematics (DESM) has involved a number of reputed experts and teachers in developing the various activities the manual presents. I wish to thank the Chief Advisor and editor of this volume Dr K. Subramanium, Homi Bhabha Centre for Science Education, Mumbai, for his contribution to this manual. We are also grateful to the members of the Advisory Committee who offered their valuable suggestions. NCERT hopes that this document will provide a useful resource to elementary schools, DIETs and other institutions

involved in teacher training programmes. If it succeeds in dispelling the fear of mathematics which is, perhaps inadvertently, inculcated during the early primary school years, its purpose will be achieved.

Director

New Delhi
23 February 2010

National Council of Educational
Research and Training

Preface

As a society, we are yet to appreciate how crucial the work of the primary school teacher is, for society as a whole. The primary teacher carries a large responsibility and is the one who can make a difference to how the child learns in the later years of school, and even to whether the child continues to be in school. Teaching at the primary level, and in particular, teaching mathematics in the first two years of school, requires a specific kind of expertise which includes both knowledge and skill. Such expertise is gained over a period of time, and gained through thinking about one's work and making a continuous effort to improve one's teaching. A teacher's manual, such as this one can only be an initial step in the long term and continuous process of the teacher's professional development. We hope that it provides some stimulation, some help, and some momentum for the teacher to move along this path.

This manual contains chapters that deal with the core topics of mathematics taught in Classes I and II. These include number and operations, shapes and space, measurement, patterns and data handling. Beside these, 'Content' chapters, there are chapters that discuss various aspects of the teacher's practice. These include topics such as using the blackboard effectively, reading and analysing the textbook, managing group work, using questions to encourage classroom discussion, paying attention to language while teaching, using open ended questions, and assessment. Thus the manual tries to support teachers in building their knowledge as well as their skill.

The chapters dealing with core topics give an overview of the topic, both what needs to be taught in Classes I and II, and what comes later. The key concepts, and the difficulties that the children may face, and ways of dealing with them are discussed. For the important topic of numbers and number operations, there is a discussion of how the children spontaneously try to solve problems involving addition and subtraction of small numbers. Children's responses, and connections of the concepts with daily life activities are discussed in all the content chapters. For

relatively new topics such as data handling and patterns, there is a discussion of the importance of these topics and the need to be included in the primary curriculum. Suggestions for activities that are motivating and strengthen children's understanding are made for all the core topics.

The chapters that focus on teachers' practices deal with some of the issues that teachers face in adopting a more learner-centred approach, as recommended by the *National Curriculum Framework -2005*. There is sufficient discussion of the details of classroom situations and what teachers can do, so that a picture begins to emerge of classroom teaching, which can provide a guideline for teachers to plan their own teaching. Practices that are highlighted or recommended are meant only to be suggestions. Teachers need to adapt, and if necessary change what has been suggested, to their own classroom context. Following the suggestions mechanically will be counter-productive.

The manual can be used in a teachers training workshop, where workshop sessions can be organised around the reading and discussion of sections of the manual. Some suggestions for exercises and questions for discussion have been included in the manual. At the same time, the manual is also designed for self-study by the individual teacher.

Group Arithmetic was constituted by NCERT to work towards improving the quality of teaching and learning of Mathematics for Classes I and II. The Group has deliberated with various content experts, pedagogists and practising school teachers and has developed this Teachers' Training Manual. I would like to thank the members of Group Arithmetic who have worked hard in brining out the draft version of this Teachers' Manual and in carefully guiding the process of publication. I am grateful to various content experts, practising school teachers and educationists who have provided valuable feedback that has improved the quality of the manual. I hope that this Manual makes a significant contribution to improving the teaching and learning of Mathematics in Class I and II.

Dr. K.Subramanium
Chief Advisor
Teachers' Training Manual
Development Committee

Teachers' Training Manual Development Committee

CHAIRMAN, GROUP ARITHMETIC

G. Ravindra, *Professor and Joint Director*, NCERT, New Delhi

CHIEF ADVISOR

K. Subramanium, *Associate Professor*, Homi Bhabha Centre for Science Education, Mumbai

CHIEF COORDINATOR

Hukum Singh, *Professor and Head*, DESM, NCERT, New Delhi

MEMBERS

Bhavna Chauhan, *Lecturer*, Gargi College, New Delhi
Deepti Behal, *Primary Teacher*, CIE Basic School, New Delhi
Dharam Parkash, *Professor*, NIE Workshop, NCERT, New Delhi
Hema Batra, CRPF School, Delhi.
Jyoti Sethi, *Teacher Fellow*, Group Arithmetic, DESM, NCERT, New Delhi
Kumud Chadha, *Teacher Fellow*, Group Arithmetic, DESM, NCERT, New Delhi
Neha Gupta, *Teacher Fellow*, Group Arithmetic, DESM, NCERT, New Delhi
Nisha Negi, USRN, JNU, Delhi
Praveen Kumar Chaurasia, *Lecturer*, DESM, NCERT, New Delhi
Preeti Chaddha, CIE Basic School, Delhi
Preeti Talwar, *Teacher*, CRPF School, Delhi
Sapna Arora, *Primary Teacher*, SKV, Malka Ganj, Delhi
T P Sarma, *Lecturer*, DESM, NCERT, New Delhi

MEMBER COORDINATOR

A K Wazalwar, *Reader*, DESM, NCERT, New Delhi

Do You Know?

According to the 86th Constitutional Amendment Act, 2002, free and compulsory education for all children in 6-14 year age group is now a Fundamental Right under Article 21-A of the Constitution.

EDUCATION IS NEITHER A PRIVILEGE NOR FAVOUR BUT A BASIC HUMAN RIGHT TO WHICH ALL GIRLS AND WOMEN ARE ENTITLED

Give Girls Their Chance !

Acknowledgement

The Council would like to extend its gratitude towards the following people and institutions:

We thank, the Homi Bhabha Centre for Science Education (HBCSE), Mumbai for providing the academic support and hosting the review workshop. Special thanks to Shweta Naik of HBCSE for sharing ideas and resources.

The Council gratefully acknowledges the valuable suggestions and feedback of Professor Anita Rampal, Professor R. Ramanujam, Dr K. Subramanium, Sh. Binay Pattanik and Smt. Phutoli Ligamir.

We also acknowledge the valuable contributions of the following participants of the review workshop who specifically helped us in reviewing the language of the manual:

Anjali Kala, *Primary Teacher*, SKV, Delhi; Ruchika Gupta, *Primary Teacher*, SKV, Delhi; Shalini, *Primary Teacher*, KV, Dwarka, Delhi; Suneeta Misra, *Lecturer*, Indian Institute of Home Economics, Delhi; Sunil Kumar, *Primary Teacher*, SKV, Delhi and Vandana, *Primary Teacher*, KV, NCERT Branch, New Delhi.

We gratefully acknowledge the following persons who attended the development workshops for the Teachers' Manual:

Arti Naidu and Radhika Dighe, Somalwar School, Nagpur; D. Kavita, Govt. Girls Hr. Sec. School, Nimbahera, Rajasthan; Bindo John, Centre Point School, Nagpur; Gurpreet Kaur, Guru Nanak Girls School, Gandhi Nagar, Delhi; Sumesh Kr.Yadav, Rewari, Haryana; Nand Kishore Verma, SCERT, Gurgaon; R.C. Singal, Gurgaon; N. Harini, *PRT*, DMS, RIE, Mysore; Laxmi Nigam, Lalitpur; Balkishan Nayak, Lalitpur; Manoj Kr. Shukla, SCERT, Uttarakhand; Kanika Sharma, *PRT*, Kulachi Hansraj Model School, Delhi; Reena Bakshi, Tagore School, Delhi; Om Prakash; N. Gayathri, DAV Public School, Hyderabad; S. Narender Yadav, Hyderabad; Amitabh Mukherjee, University of Delhi; Harmesh Lal. Delhi; Anjali Norauha, Eklavya, Bhopal; Kamal Kant Swami, Rajasthan; A.Ram

Prasad, Andhra Pradesh; Sunder Dass Dhamija, Fatehbad, Haryana; Y. Venkat Reddy, Andhra Pradesh; Professor I.K. Bansal, DEE, NCERT; Sartaj Singh, *PRT*, DMS, RIE, Ajmer; Sajith Gopinath, NID, Ahmedabad; B.R. Diengdoh, Meghalaya; Raju Borkar, *PRT*, DMS, RIE, Bhopal; Ghanshyam Patra, *PRT*, DMS, RIE, Bhubaneshwar; Asmita Varma, Navyug School, Lodhi Colony; P.B. Lartang, DERT, Shillong; Swarn Lata Sharma, KV, Moradabad; Donnipad Manjunath, KV, Ojhar, Maharshtra; Shobha, *Assistant Teacher*, Uttarakhand; Dr. Sanjay Dhyani, *Lecturer*, DIET, Ratura, Uttarakhand; Nafe Singh Punia, Nidara, Harayana; Binay Pattanayak, *Chief Consultant* (Pedagogy), Tech. Support Group (SSA), New Delhi.

The Council acknowledges the efforts of Surender Kumar, Ajeet Kumar Dabodiya, *DTP Operator*, DESM, NCERT, New Delhi.

Contents

1. Numbers 1-23

2. Addition and Subtraction 24-35

3. Shapes 36-45

4. Measurement 46-62

5. Data Handling 63-77

6. Patterns 78-88

7. How to Assess Children's Progress 89-96

8. Open Ended Questions in Mathematics 97-104

9. Teacher's Questions and Classroom discussion 105-112

10. Managing Group Work 113-122

11. Using the Blackboard 123-131

12. Analysing Textbooks 132-138

13. Language and Mathematics Learning 139-148

Gandhiji's Talisman

I will give you a talisman. Whenever you are in doubt or when the self becomes too much with you, apply the following test:

Recall the face of the poorest and the weakest man whom you may have seen and ask yourself if the step you contemplate is going to be of any use to him. Will he gain anything by it? Will it restore him to a control over his own life and destiny? In other words, will it lead to Swaraj for the hungry and spiritually starving millions?

Then you will find your doubts and your self melting away.

mkGandhi

1

Numbers

Children see numbers being used in a variety of situations around them. They themselves use number in different situations. Like, I want to eat two *chapattis*, I came 2nd in the race, I go home in bus number 5, I am 6 years old, Give me Rupees 10 to buy a colour box, there are two 'chairs'.

In these situations the numbers may have very different meanings.

For example –

- When we say there are 5 children in a group, **'5'** represents **quantity**– how many children there are in the group. So if there are two groups, one with 5 children and other with 6 children then we can add the number of children in both the groups.

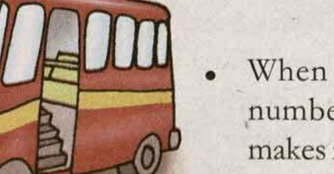


Fig. 1.1

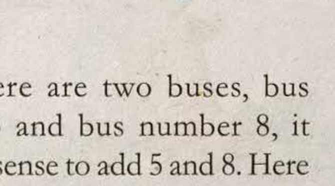


Fig. 1.2

- When there are two buses, bus number 5 and bus number 8, it makes no sense to add 5 and 8. Here number **'5'** and number **'8'** are only used as **labels** for bus routes.

Fig. 1.3

Fig. 1.4

- When we say that a person is **5th** in a row then we are talking about **order** of that person in a row.
- In situations where we say **'5 years old', '3 litres of milk'** or **'100m race'**, unlike in the counting situations, the numbers may not correspond directly to what the child can see. These numbers indicate **measures** that are more difficult to understand than numbers used for counting.

Children learn to distinguish these different uses and meanings of numbers as they grow older and gain more experience.

When children enter Class I they have intuitive ideas about numbers and quantities, which they have learnt from their surroundings or from their experiences. For example, if a pre-school child is asked to choose between the toffees in two hands she prefers to choose the hand with more toffees. She may also know a few counting words. She may say," I want ten toffees", even though she may not understand the quantity that the number ten represents. Children whose brothers or sisters are involved in buying and selling may know something about money.

Children first use 'number words' in counting by reciting which is not really counting. For example, children recite number rhymes like, *One-two buckle my shoe, three-four shut the door*... they practice the number sequence by reciting. Later on they use the number words also for counting objects.

. 1.5

In Classes I and II they learn to say counting words up to hundred and learn to read and write numerals. Gradually, they develop reasonable skill with counting and learn the pattern of tens and ones in the number words.

Further they develop a sense of numbers up to hundred. They also learn to do addition and subtraction on numbers up to 20 and use their knowledge to solve problems in different contexts. For example, on his birthday party, Rohit, a Class I child invited 9 of his friends and got a return gift for everybody. If only 7 friends came for the party, without counting the number of gifts left he can tell us that if seven friends came to the party then two gifts should be left. Similarly, if a child while playing Ludo, needs 9 to win the game then she might say that she needs 6 on the first throw and 3 on the second.

It is not very convenient or useful to strictly draw a separating line between what is done in Classes I and II. At this age, children progress at different rates depending on their initial knowledge, motivation and abilities. A child who is behind her classmates in Class I, may catch up in Class II. So, Classes I and II form a continuum of learning. What children learn in Class II is an extension of what they have learnt in Class I.

In Class II they revisit addition and subtraction with numbers up to 20 and solve problems. Over the year, they revise and strengthen their knowledge of numbers up to 100. They learn different ways of adding and subtracting by drawing representations of tens and ones without and with regrouping. They are also introduced to the vertical column algorithm for addition and subtraction.

As children move from Class I to Class V, they learn and choose different ways of doing addition and subtraction to find out whether the results are reasonable or not besides learning the standard methods. They also learn the operations of multiplication and division. They develop the ability to deal with numbers mentally and to think about numbers without using objects. Like, for adding 3 pebbles to 4 pebbles, a child might

Fig. 1.6

Fig. 1.7

Fig. 1.8

feel the need to take 3 pebbles first and then 4 more, put them together and count all of them to know the answer, or she migh do it by drawing strokes on paper, or she might start from and then count three more on fingers to find the answer, o might directly retrieve the answer from her memory.

As they move to Class III and beyond, they move from using objects to working with symbols. With time they learn to deal with big numbers as well. For example, for doing 153 +27 in class, a child makes use of the method taught by adults o writing numbers in columns, adding ones to ones, then tens t tens taking the 'carryovers' and then hundreds to hundreds.

Fig. 1.9

Another child might discover her own strategy. She migh add hundreds first i.e. 100 and 200 is 300 and then tens i.e. 5 and 70. For that she might first say 50 and 50 is 100 and 70 is 2 more than 50 so it makes 120, that makes it 420 and now 3 an 3 is 6. So, the answer is 426. It is important to encourage children to use mental computation and find their own ways of solving problems and estimating the result. Only by doing so will children develop a feel for numbers, and develop efficient and accurate strategies for computation. Teachers can help students understand numbers and number operations by discussing the representations and strategies they use.

But before all this, in Classes I and II the teacher needs to help children lay a sound foundation of counting, moving on to place-value and number operations.

Fig. 1.10

COUNTING

We often see young children saying counting words while playing climbing stairs or collecting their toys and other things from thei surroundings. Children are fascinated or attracted by numbers. They try to copy other children or adults who use counting words. They use the counting sequence in the rhymes they sing o the games they play. For example, the normally start a race by saying, "one, two, three and go!" Or when they play "*chupaan chupaee*" ("I spy" or "hide and seek") the child who is spying others counts up to 10 or 20 to give others a chance to hide.

A few rhymes which children usually like are

One, two, three, four, five
Once I caught a fish alive
Six, seven, eight, nine, ten
Then I let it go again
Why did you let it go?
Because it bit my finger so.

Hindi rhyme...

हम हाथी को गिनती सिखाएंगे,
हम हाथी को गिनती सिखाएंगे।
हाथी बोलो एक,
तुम्हें मिलेगा केक।

थोड़ा तुम खाओगे, थोड़ा हम खाएंगे,
हम हाथी को गिनती सिखाएंगे।
हाथी बोलो दो,
तुम्हें मिलेगा ढोल।

थोड़ा तुम बजाओगे, थोड़ा हम बजाएंगे,
हम हाथी को गिनती सिखाएंगे।
हाथी बोलो तीन,
तुम्हें मिलेगी बीन।

थोड़ा तुम बजाओगे, थोड़ा हम बजाएंगे,
हम हाथी को गिनती सिखाएंगे।
हाथी बोलो चार,
तुम्हें मिलेगी कार।

थोड़ी तुम चलाओगे, थोड़ी हम चलाएंगे,
हम हाथी को गिनती सिखाएंगे।
हाथी बोलो पाँच,
करना पड़ेगा नाच।

थोड़ा तुम नाचोगे, थोड़ा हम नचाएंगे
हम हाथी को गिनती सिखाएंगे।

(Reference: Department of Human Development and Family Studies, college of Home Sciences, Maharana Pratap University of Agriculture and Technology, Udaipur)

Most children learn counting words early but they may not associate number with quantity. For example, Neha, a 2-year old child can recite numbers up to 10 easily but cannot make use of her knowledge of the number sequence for counting objects. When asked to give 6 toffees she gave whatever she could pick at one go.

Why did Neha do this?

____

____

____

There may be different reasons for why Neha could not give 6 toffees. We will explore these in more detail in later examples. Let us list some possible reasons briefly.

- Neha may not know that counting must be used to gather 6 toffees.
- She may be unsure about her knowledge of the sequence of counting words.
- She may have a difficulty in associating a number word with every toffee while counting.
- Perhaps she has a difficulty in arranging toffees in such a manner that she can count them without skipping any toffee or counting any toffee twice.

It is therefore important to distinguish whether Neha is only able to recite number words correctly, or she can make use of her knowledge of counting to count objects as well. Reciting counting (i.e. reciting number names in sequence) is an activity that is generally learnt by the children before using counting to count objects and that is what Neha is doing. She knows counting but does not know its function. In this case, the function o counting is to quantify the toffees with the help of the process of counting i.e. making one to one correspondence of objec and the number name to gather 6 toffees in all.

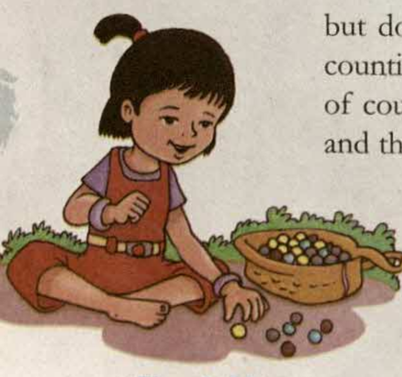


Fig. 1.11

Counting involves many skills or processes. We are so accustomed to these processes that we use them withou realising that we are doing so. To analyse these processe let us imagine the following task:

There is a bag full of different coloured beads, you have to count and make a string using 9 yellow beads

The normal course of doing this task would be as follows. However as adults we skip or do not notice many of these steps due to our proficiency in counting.

- Identify the yellow beads, which we want to use.
- Sort them from the remaining beads, which are of different colour or shape.
- Arrange the beads in such a manner or order that we don't skip one or count one bead twice.
- Once they are ordered, say some words: one, two, three and so on.... It is essential that we know these words in the fixed order.
- As we recite these counting words, we use one number word for each bead, saying as many number words, as there are beads. This process of counting is called **one-to-one correspondence**.
- In this case 9 is the word we say as we touch the last bead. We conclude that there are 9 beads, which will go into the string. That is, we have selected the last number word i.e. 9 that we recited and associated it with the whole group. This act of associating the quantity of the whole group with the last number recited is called the **cardinality principle.**

Children follow these steps and often make errors. To illustrate, a teacher gave 8 counters each to 5 students of Class I to count and obtained the following responses.

Let us analyse what each child did and discuss how we can help him/her in the process of counting.

(1) Mohit kept the counters in a circle and while counting, counted 3 counters again and said there were 11 counters. At times children don't arrange objects in an order so that they don't skip or recount some objects again.

As teachers we can help children in arranging objects so that it is easy to count and can prompt them to do so themselves. We need to bring the child's attention to the fact that each item is to be counted only once.

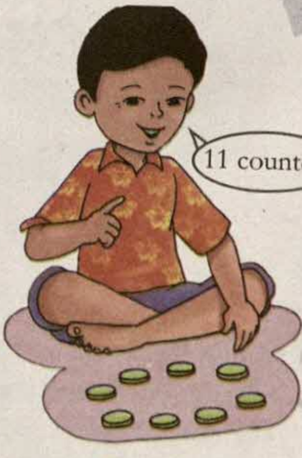


Fig. 1.12

(2) Khushi recited the counting words as "One, two, three, five, six, seven, eight, and ten." She associated each counting word with a counter but missed out reciting 'four' and 'nine'. She

Fig. 1.13

has constructed her own counting sequence and the imperfections in it are not important to her. We can help Khushi by giving her practice with the number sequence or counting words through playing games or through rhymes. Children enjoy reciting 'one, two, three, and four' and so on and we can encourage them to recite the sequence as often as they like. One of the games which children enjoy playing is 'I say one and you say two, I say three you say four and so on.'

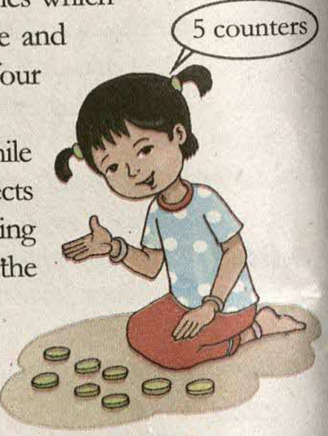


Fig. 1.14

(3) Kiran said there were 5 counters. While counting she was touching the objects faster than she could say the counting word. So, by the time she touched the eighth object, she was saying five. She could not pair each counting word with an object.

(4) Ali counted till 8 but could not say how many counters there were. When asked again, he again started 1, 2, 3 and stopped at 8. For him 'how many' means counting till you have reached the final object. He could not associate the final counting word with the whole group.

Fig. 1.15

We can help children who are unable to relate the final counting word with the whole set by creating different opportunities for counting. Like, engaging children in games such as board games, stories and rhymes.

The following rhymes highlight the use of a number word to say 'how many'.

One little rabbit wondering what to do;
One more came along – then there were two.
Two little rabbits sitting down to tea;
One more came along – then there were three.
Three little rabbits knocking at the door;
One more came along – then there were four.
Four little rabbits going for a dive;

One more came along – then there were five.
Five little rabbits, getting up to tricks;
One more came along – then there were six…

One little kitten, Went to the zoo
Along came another that made two
Two little kittens climbed up a tree
Along came another that made three
Three little kittens, looking for some more
Along came another that made four
Four little kittens, calling for their mother
Now they are five, and all are together.

(Reference: Maths for Every child, Class I, Homi Bhabha Curriculum for Primary Mathematics.)

आओ गिनती गायें

एक मोटा हाथी झूम के चला
मकड़ी के जाल में जा के फँसा
जाल को देखा देख के डरा
दूसरे हाथी को इशारे से बुलाया

दो मोटे हाथी झूम के चले
मकड़ी के जाल में जा के फँसे?
जाल को देखा, देख के डरे
तीसरे हाथी को इशारे से बुलाया

तीन मोटे हाथी झूम के चले
मकड़ी के जाल में जा के फँसे
जाल को देखा, देख के डरे
चौथे हाथी को इशारे से बुलाया

चार मोटे हाथी झूम के चले
मकड़ी के जाल में जा के फँसे
जाल को देखा, देख के डरे
पाँचवे हाथी को इशारे से बुलाया

पाँच मोटे हाथी झूम के चले
मकड़ी के जाल में जाकर फँसे
जाल को तोड़ा, तोड़ के हँसे
हँसते-गाते घर को चले।

(Reference: Department of Human Development and Family studies, college of Home Sciences, Maharana Pratap University of Agriculture and Technology, Udaipur)

Words like 'made', 'all together' promote the idea of cardinality.

If counting is taught as one more at each step then it helps children understand that eight means set of eight objects and not just eighth object.

Some games that can promote the concept of cardinality, also which children enjoy playing are

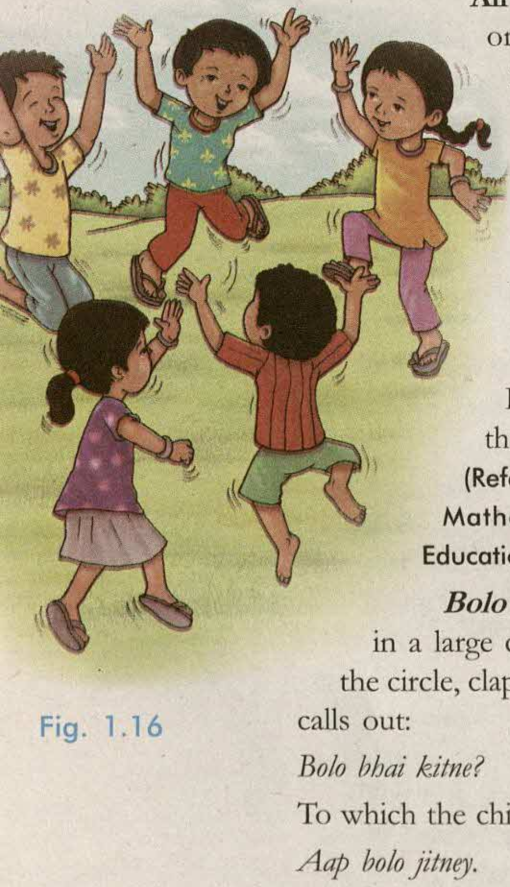


Fig. 1.16

All together now—In this game the teacher or one of the students gives instructions to be followed by the children who are standing in their places or in a circle.

Clap two times!

Take three steps back!

Jump four times!

Number game—In this game teacher or a student says a number and children have to jump or clap as many number of times. Like if the leader says three, all the kids jump three times etc.

(Reference: *Games and Activities* for Class I, School Mathematics Project (1996). Centre for Science Education and communication, University of Delhi, Delhi)

Bolo bhai kitne—Teacher makes the children stand in a large circle (they could be slowly moving round in the circle, clapping). The teacher or one of the students then calls out:

Bolo bhai kitne?

To which the children reply—

Aap bolo jitney.

The teacher can then call out a number, say, 5. The children now form groups of 5. Children who fail to do so are "out". The process can now be repeated with the teacher or student calling out a different number.

(Reference: *Maths for Every Child*, Class III, Homi Bhabha Curriculum for Primary Mathematics.)

Board games like 'Snakes and Ladders' or 'ludo' can also be used. When a child throws dice and sees 5 dots on it and moves her peg 5 steps ahead then she is translating the number word into a counting action.

Fig. 1.17

Celebrating number —A day is spent in celebration of a particular number: say, a day of four. Teacher can tell children in advance to collect objects or pictures which have some "fourness" in them. Like a picture of a car with four wheels, a twig with four leaves on it etc. On the appointed day children get the material to the class. They can work individually or in groups to make posters, drawings, poems, stories, etc. in the classroom. In the end they can be encouraged to display or recite whatever they have come up with in front of the class.

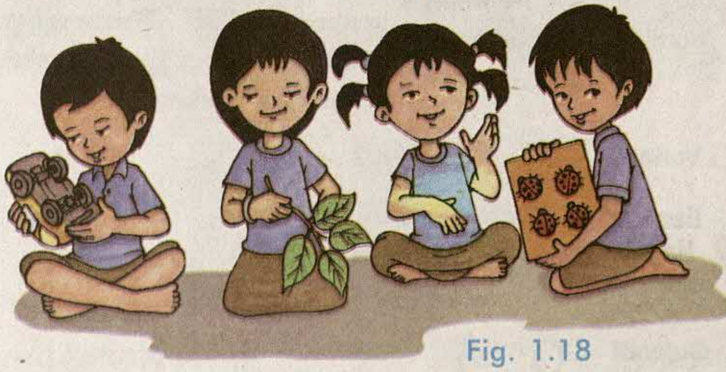
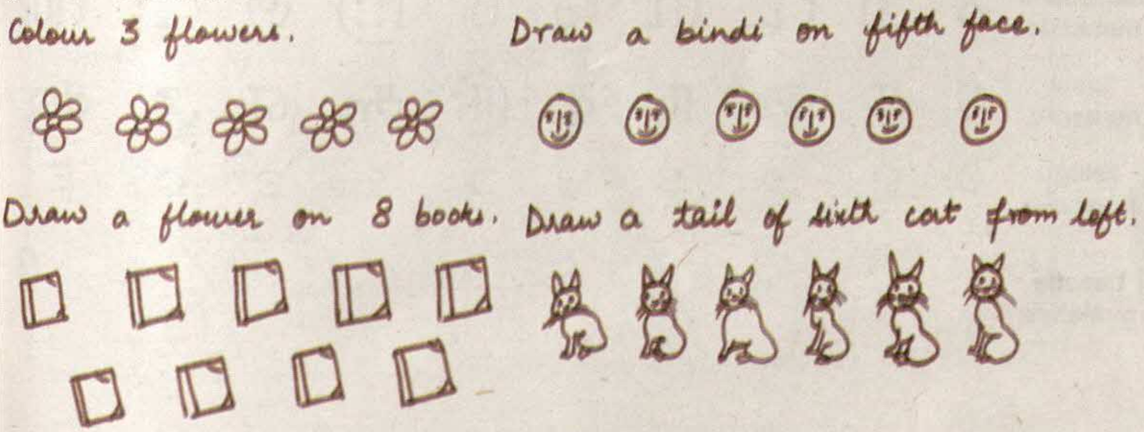


Fig. 1.18

Worksheets

Count and colour worksheets with rows of different objects with instructions to colour say, 1 pencil, 3 cups, 3rd cat, etc., can be given to students.



Colour 3 flowers.

Draw a bindi on fifth face.

Draw a flower on 8 books.

Draw a tail of sixth cat from left.

Fig. 1.19

Note for Teachers

Teachers please note that terminology such as 'numeral' is given just for your understanding. While teaching please do not expect the children to understand the distinction between 'number' and 'numeral'.

NUMERALS

Numbers can be shown or represented in different ways. The common representations that young children learn are 'spoken number words' and written 'numerals'. (Besides these, a less common representation is tally marks).

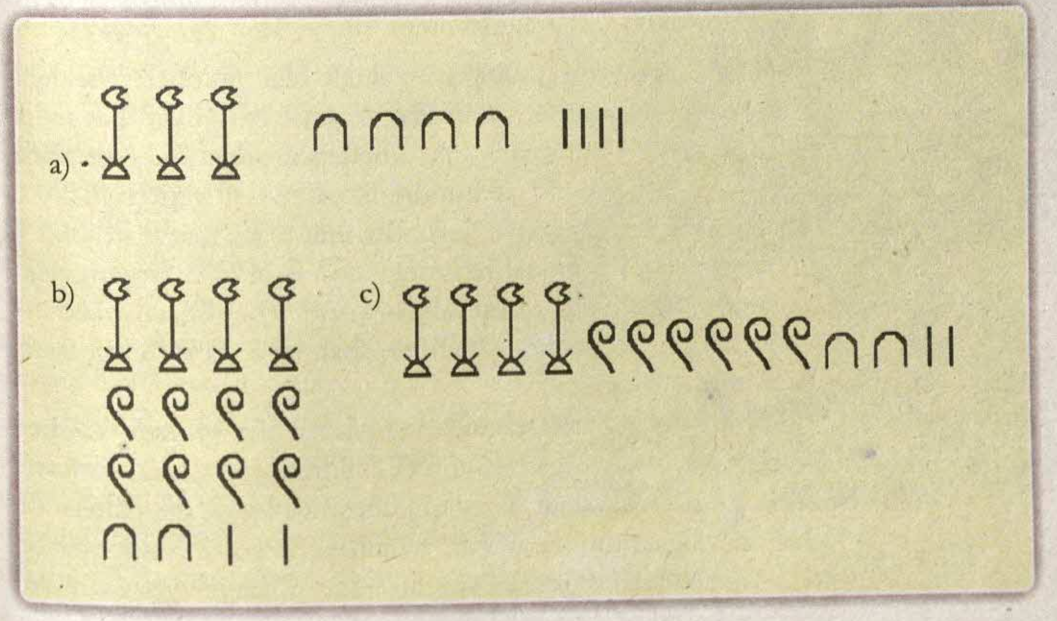
The written symbols that we use to represent number names are called numerals. In the Devnagari script the numeral for the number **thirty-five is "३५"** while in the International numeral system it is "35". **Numerals change according to the script but the number that is represented remains the same.** Look at the chart below.

Variant	0	1	2	3	4	5	6	7	8	9
Eastern Nagari numerals	০	১	২	৩	৪	৫	৬	৭	৮	৯
Gujarati numerals	૦	૧	૨	૩	૪	૫	૬	૭	૮	૯
Gurmukhi numerals	੦	੧	੨	੩	੪	੫	੬	੭	੮	੯
kannada numerals	೦	೧	೨	೩	೪	೫	೬	೭	೮	೯
Malayalam numerals	൦	൧	൨	൩	൪	൫	൬	൭	൮	൯
Tamil numerals	௦	௧	௨	௩	௪	௫	௬	௭	௮	௯
Telugu numerals	౦	౧	౨	౩	౪	౫	౬	౭	౮	౯
Lepcha numerals	᱀	᱁	᱂	᱃	᱄	᱅	᱆	᱇	᱈	᱉
	0	1	2	3	4	5	6	7	8	9

Other number systems with different symbols were developed in the past in different countries. The Egyptian number system used the following symbols to represent numbers.

Value	1	10	100	1,000	10,000	100,000	1 million, or infinity
Hieroglyph	𓏺	𓎆	𓍢	𓆼	𓂭	𓆐 or 𓆏	𓁨
Description	Single stroke	Heel bone	Coil of rope	Water lily (also called Lotus)	Finger	Tadpole or Frog	Man with both hands raised

So, the numeral in the Egyptian system for 3044 is as shown below in (a)



Here the symbol for 1000 is written three times to represent three units of 1000 or 3000. Note that changing the order of the symbols does not change the number, because the symbol for a unit of 1000 does not depend on the position, but on the shape.

Since the symbols for different units can take different positions, position does not give additional information in the Egyptian system.

Egyptian numerals are written in both the directions, vertically as shown above in (b) and is written in left-to-right as shown in (c).

The **Hindu-Arabic system or the decimal system**, which we follow, is a **positional system**, i.e., the position of the digit indicates which unit it represents. So we cannot change the position of a digit without changing the number.

For example

What numbers do the digits 1, 4, 2, and 5 represent in 1425?

____

____

____

The digit 1 designates the number of units of 1000 or thousands, 4 the number of hundreds, 2 the number of tens and 5 the number of ones.

As far as we are dealing with single digit numbers, the digit represents the number itself. For example, the digit 9 at units place represents number 9 only, whereas the digit 9 at tens place will represent 90 and at hundreds place will represent 900. Therefore when we move beyond a unit place that is beyond 9 understanding positional representation is of great importance. To understand the numerals beyond 9, children need to understand place-value, which we shall take up for discussion later in this chapter.

Note that to understand or to learn the spoken number words like 'ten' or 'twenty-three' children don't need to learn about positional value. They only need to know the pattern of the two digit number words, which is based on multiples of ten. This pattern is different in different languages. For example, in North Indian Languages, the units are indicated before the tens (तेईस). There is a special pattern for numerals like 29(उनतीस), 39 (उनतालीस) etc.

RECOGNISING AND WRITING NUMERALS

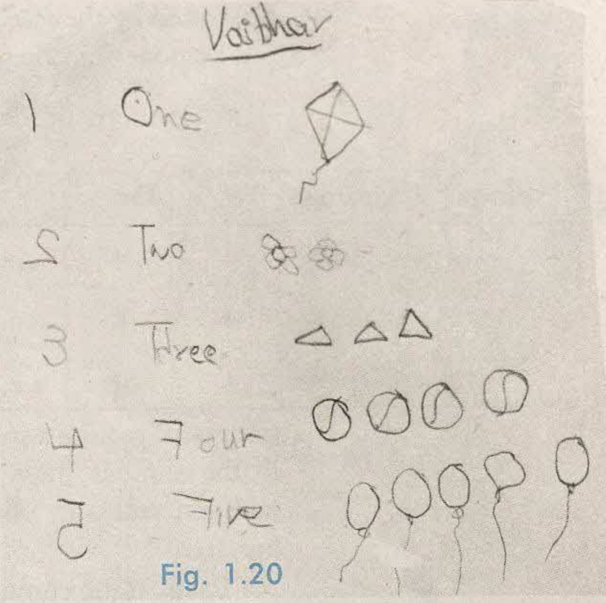

Vaithar
1 One
2 Two
3 Three
4 Four
5 Five

Fig. 1.20

The numerals up to 9 are simple, but writing them still requires considerable control and coordination. We often come across notebooks with numbers written as in the figure.

Many children write numerals like their mirror images. It is normally one of the biggest concerns of the teachers as well as the parents. But, is it really such an important area that a lot of time and energy is invested in it? Suppose that a child writes '7' as '٢' but gives us seven objects when asked, is it not acceptable?

A child should understand that '7' means a set of seven objects. In time, with sufficient practice and gentle reminders from teachers or parents, children grow out of their habit of writing 'mirror image' numerals.

A teacher once consciously wrote the mirror image of three on board and asked children what she had written. Children giggled over the mistake committed by their teacher and also came to the board to correct her.

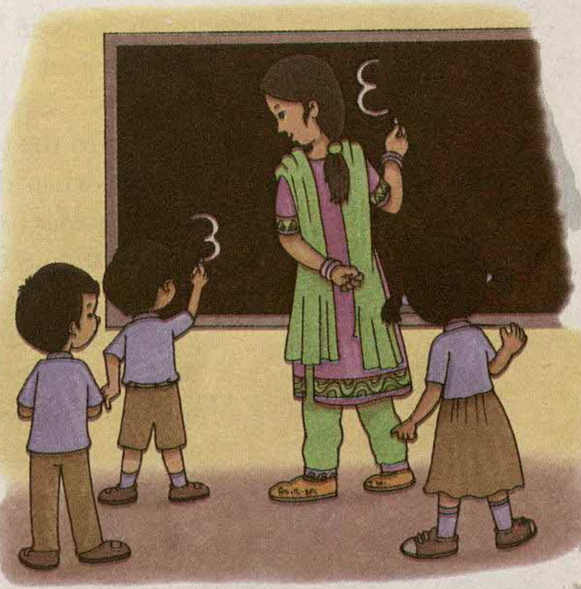


Fig. 1.21

Children see numerals around them constantly – on house-doors, buses, car number plates, clocks, etc. Learning the numeral becomes more meaningful for children if they are familiar with the number. After doing many activities with concrete objects like making a set of 4 pencils, 4 pebbles, 4 leaves, 4 flowers, 4 children, etc., the symbol '4' can be written by the teacher on the board or paper. Such activitie can be done for other numerals as well.

The Concept of Zero

ousands	Hundreds	Tens	Ones	
	3	4	4	i.e. 344
3		4	4	i.e. 3 44
3	0	4	4	i.e. 3044

If we look at the Egyptia numerals shown in the previou section carefully, we realise tha the system does not have a symbol for zero. In fact the zer symbol is not required in thi system. However, in a positiona system we require a special symbol for the **empty placeholder**. In the number 'three thousand and forty four i.e. 3044", the numeral will only show thousands, tens and ones, but 'no hundreds'.

To fill in the empty position, we use the symbol zero. If we did not have a zero symbol then we would not be able to distinguish between 'three thousand and forty four' and 'three hundred and forty four'. So we write 'three thousand and forty four" as 3044. Here **zero is playing the role of the empty placeholder in the position for hundreds.**

Fig. 1.22

Similarly, when the numbers contain no units of tens, we need a placeholder for the place of tens. In our system, the **Zero** symbol i.e. **0** functions as the **empty placeholder**. This placeholder helps us to distinguish 11 from 101.

Besides being a symbol for the empty place, **zero** is also a **number**. When we say that the freezing point of water is 0 degree Celsius or refer to the point zero on the number line, then zero is not a placeholder but a number.

Since zero changes its role, it confuses children. Initially it is better to emphasise the use of zero to indicate the absence of a particular thing. For example, 'zero children' means 'no children', 'zero flowers' means 'no flowers', zero hundreds means 'no hundreds'.

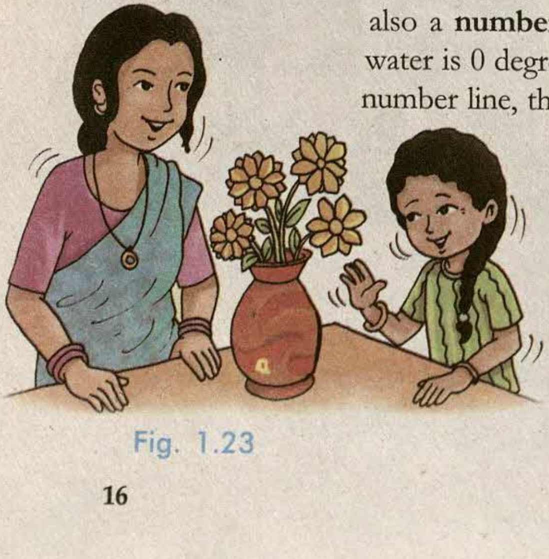


Fig. 1.23

A teacher wanted to introduce the concept of zero to her class. She took a flower vase with 4 flowers and asked the children to count how many flowers there were?

Student: 4 flowers

(She pulled out one flower from the vase)

Fig. 1.24

Fig. 1.25

Teacher: Now, how many flowers are there?

Student: 3 flowers

(She kept pulling the flowers from the vase until no flower was left.)

Teacher: What is left?

Student 1: vase

Student 2: nothing

At this point if the teacher says, "zero flowers are left", it would be less confusing than if she says, "zero means nothing is left". We often introduce zero as **nothing** whereas **it is preferable to explain it as absence of a particular thing.** It is better to let children experience zero using a variety of concrete objects.

PLACE-VALUE

The two digits in the numeral 11 have different values because of their places. But when teaching children, we normally say one and one eleven; one and two twelve and so on. This is mathematically wrong as well as confusing.

Tens	Ones
1	1

Let us look at the number 11, an example which often confuses the children.

What is the place-value of 1 in the number 11?

The right-hand digit has a place-value of one, whereas, the left-hand digit has a place-value of one ten. So eleven is "one ten and one".

So it is correct to say "one ten and one is eleven" rather than "one and one is eleven."

Base 10

Our number system is based on the number ten. Using only ten digits (0 to 9), we can represent any number, however small or large.

It is a base 10 system, that is, it is based on grouping of ten: ten ones make one ten, ten tens make one hundred, ten hundreds make a thousand and so on.

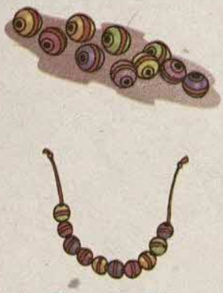


Fig. 1.26

Write the largest number you can think of.

The word 'ten' may represent a single entity (like one necklace having ten beads) and, at the same time, ten separate units (like ten loose beads,), namely, ten ones. These representations are interchangeable. When we introduce a base in the number system, it becomes possible to show larger numbers and calculations become easier. Ten was chosen as the base probably because we have ten fingers, which are 'handy' when we need to count.

Roman numerals do not employ place-value; calculations are difficult to perform using these numerals.

Try adding XXIX and XLIV

Fig. 1.27

In the classroom we normally make children write 0 to 9 many times and then we introduce 10 to 19 and make them practice these numbers for some time and then go on to 20 to 29, and so on.

In one such class the teacher asked the children to write the counting numbers from 0 to 50. A child wrote 0 to 9 first and then 1, 1, 1, and added 0-9 to those one's to make 10 to 19 and so on.

What do you think about the shortcut followed by the child?

We as teachers need to draw the attention of the children to the fact that '2' in the number 23 indicates 2 necklaces of ten and '3' indicates three more ones (loose beads). For making children understand this, we can take up the concept of 'grouping'.

GROUPING

We follow the base ten system that is based on grouping of ten. It is important to let children group objects in 2's first since it is easier for children to handle small groups. It also gives them a hint that a group of two is different from loose single objects. So, if I have 2 groups of two beads each and 1 loose bead then, it does not mean that I have 3 beads but 5 beads. Children need sufficient practice with counting grouped objects properly.

Note for Teachers

Other local songs can also be used instead of 'fire in the mountain', you can also substitute the rhymes given in this chapter from those rhymes which are part of the child's local language.

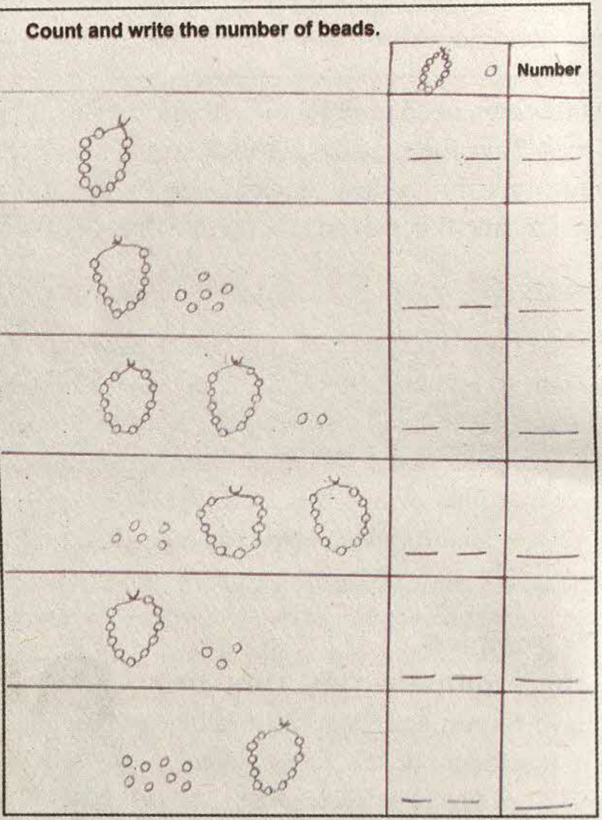
Games for grouping

Fire in the mountain run, run, run: children have to run and then the teacher calls out a number, say '5'. The children have to make groups of 5. Those who fail to do so will be out of the game.

Freeze game is a variation of the above game. In this children are supposed to move till the music or drumbeats are on. Once the music is turned off, they form a group of the number decided upon earlier and freeze.

Fig. 1.28

Clicks and Claps: An interesting game for grouping by ten is clicks and claps. First you need to explain to the children that 10 claps are equivalent to 1 click. So, for 18, they can click once and clap 8 times or else they can clap 18 times. They can have fun showing numbers using clicks and claps, and also converting one to the other. If the children find it difficult to click with their fingers, they can say 'tick'.



Count and write the number of beads.

		Number
	___ ___	___
	___ ___	___
	___ ___	___
	___ ___	___
	___ ___	___
	___ ___	___

Making necklaces: For grouping, beads or counters can be used. Initially, a set of 10 beads can be joined together to make 1 ten or a necklace.

This representation of a ten can be used in worksheets with pictures like the following.

The concept of 13 as consisting of both, a bundle of ten sticks and three loose sticks, and as 13 loose sticks, should be reinforced. We can make tens and ones with any of these objects taken from the child's immediate environment.

- Ice cream spoons
- Matchsticks with the sulphur removed by soaking in water and rubbing with a cloth.
- Broomsticks can easily be bundled by tying with a rubber band.

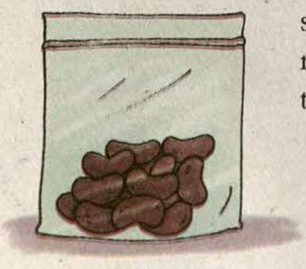


Fig. 1.29

- Beads can be put on a string, which can be tied to make a necklace.
- *Rajma* seeds can be put in small packets.
- *Bindis* can be stuck on a card and can be used.
- Some children are familiar with money even at a young age. If this is so, one can use ten rupee notes and one rupee coins to indicate tens and ones.

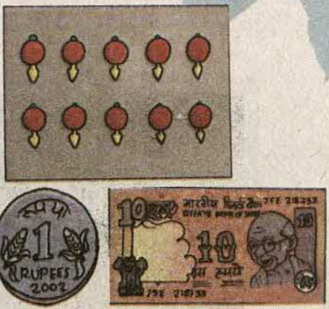


Fig. 1.30

- Blocks given in the learning kit can also be used. Children can be provided with the blocks and then asked to make **towers out of ten blocks**. Let them make a few towers and then the teacher can ask how many loose blocks make a tower? 2 towers and so on?

Fig. 1.31

Teacher can also ask how many towers (of ten) they can make out of 15 blocks. How many loose blocks will be left?

Material like blocks, bundles, *bindi*-cards, etc., help students express 24 as 24 ones (units), 1 ten and 14 ones, or 2 tens and 4 ones.

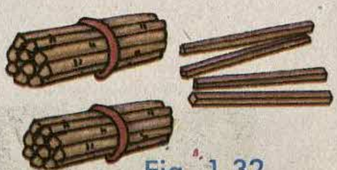


Fig. 1.32

If children understand the principle of grouping of objects by ten, this will help them understand and use place-value when they write the numerals beyond 10.

Writing of numbers

As children understand the grouping based on ten, they need to use this to read and write numerals beyond ten. We have seen several examples of how we can show tens and ones using concrete representations. However, the conventional representation using numerals is different from all of these. To understand the difference, see the representations of 23 shown in the figure here.

Library
04.07.2011
14562

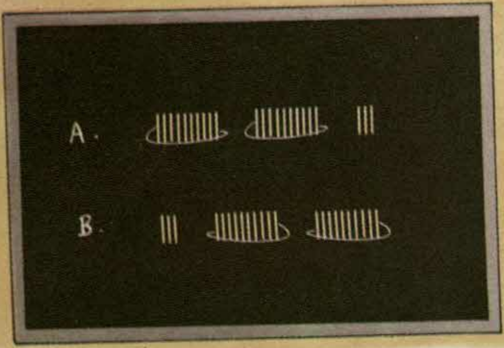


Fig. 1.33

Does change in concrete representatio from figure A to figure B change th number?

Now if we write the numerals belo the tens and ones and read only th numerals will they change from figur A to figure B?

We see that changing the position o the concrete representations for tens an ones does not change the number (reca the Egyptian numerals).

However, while writing the numerals the arrangement of the digits matters. I the numeral the digit for the tens must b written to the left of the digit for the units When we interchange the digits in the numeral, the number changes from 23 to 32, unlike in the representation using pictures of concrete material.

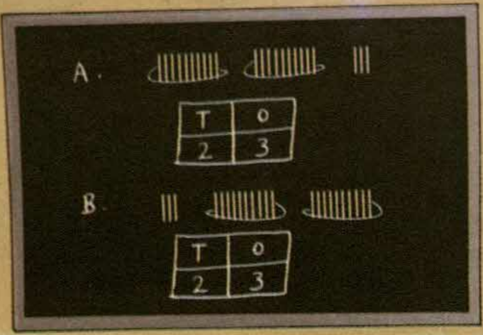


Fig. 1.34

Children need practice and help wit decoding the units in the numeral base on position. We can help children i writing numbers by initially asking them to make drawing o tower with ten blocks and 1 loose block and under that writ the number of towers and loose blocks. While reading th numeral, ask the children which digit shows how many tens ther are and which digit shows how many ones there are.

Attention needs to be given especially to the interpretatio of zero in the units place. Children often write 207 when tol to write 'twenty-seven'. To help them, we can use **arrow card** and can show them that 20 and 7 is 27. Arrow cards are used t show the expanded notation of number.

There are three types of cards in one set of 'arrow cards' The first type is the smallest size card which represents 'units'

There are 9 cards for each unit from 1 to 9.

The second type of card is bigger than the 'units card'. This card is to represent 'tens'. There are 9 cards such as 10, 20, 30, 40, 50, 60... up to 90. Similarly the third set represents 'hundreds' such as 100, 200, 300... upto 900.

For example: To represent the number 564, child needs a 'unit' card of '4', a 'tens' card of '60' and a 'hundreds' card of '500'.

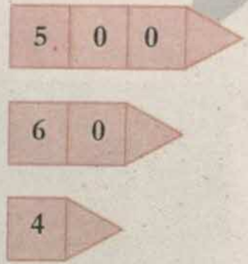


Fig. 1.35

Such a representation shows the expanded notation of a number and hence helps in resolving the confusion because of the zero.

Other than 'Arrow Cards' children can also use 'play money' and depict the number. Similarly they can show the number with the blocks and vice-versa.

Discuss and work out any other activity that can be used for explaining the place-value in terms of position.

____

____

In this way we can ensure that children understand the value of a number and do not just treat the number as a mere symbol. It is important that children understand counting and do not just rote learn the sequence of number words. It is also important that children understand grouping and place-value so that they can understand the number with its value.

Addition and Subtraction

2

Problems involving operations such as addition and subtraction allow the children to move beyond counting concrete objects to more abstract uses of number. For example, I have some marbles in my left hand and some in my right hand. The child counts the ones in my left hand–5, and counts those in my right hand–3. Then I put all the marbles together and ask her how many are there. She again counts all the marbles and says 8.

But I could modify this task in the following way. I ask how many marbles are in my left hand? 5. How many are in my right hand? 3. Then I put the marbles together, close my hand and ask the child how many marbles are there in all. Now it is not possible for the child to count the objects in my closed hand, but she needs to solve the problem using some abstrac representation. She may use her fingers, o other objects, or count mentally. Thus addition tasks push the child to mov beyond concrete counting towards the us of abstract representations.

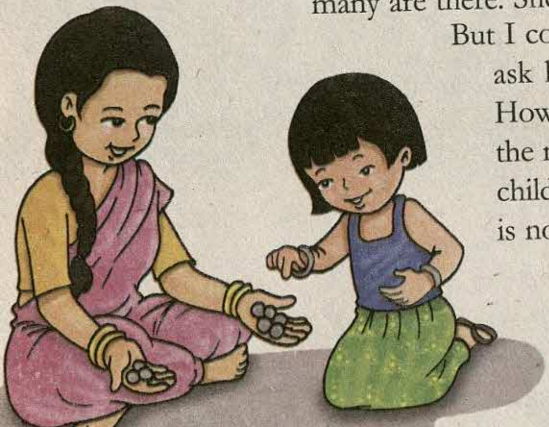


Fig. 2.1

One can pose different addition tasks, which require the chil to make her own abstract representations, using concrete object or pictures or by describing a situation in words. Some mor examples are given below. It is helpful if these 'story problems are presented using concrete objects or pictures as suppor This will help the child imagine the situation.

(a) Rama made 5 paper caps. Her mother made 3 more. How many caps does she have now?

(b) There were three rooms in the school. Three more were constructed. How many rooms are there now?

Fig. 2.2

Fig. 2.3

(c) Two children were playing marbles. Five more joined them. How many children are playing now?

(d) Three of Reena's friends are girls and four are boys. How many friends does Reena have in all?

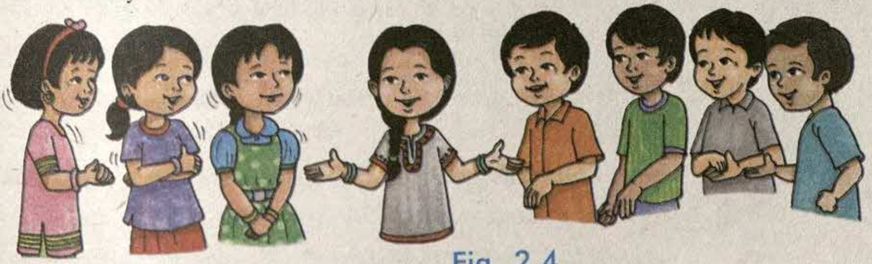


Fig. 2.4

(e) In a room, five people are standing and three are sitting. How many people are there in all?

(f) Rajeev was playing ludo with his friend. On the first throw he got 6 on the dice, on the second, he got 3. How many boxes should he move ahead?

Children used **different representations** to solve these problems.

Fig. 2.5

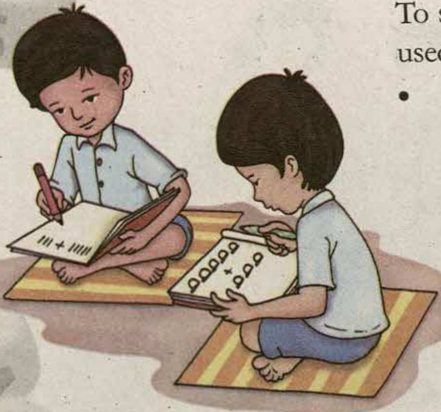


Fig. 2.6

To solve the first problem (5 caps + 3 caps) children used the following ways:

- Mohit used concrete material to solve the problem. He picked up 5 blocks and kept them aside. Then he picked up 3 more blocks and added those one by one to the 5 blocks he had kept aside, saying 6, 7, and 8.
- Another child, Shahbaz drew pictures of circles to solve the problem.
- Hemant made tally marks and counted them.
- Niyati opened five fingers of one hand and three of the other and counted all the fingers to find the answer.

All four of them used different representations for solving the same problem. Initially children may use objects, tally marks, pictures or fingers to solve problems and gradually start solving them mentally.

Fig. 2.7

Situations for Addition and Subtraction

The situations, which commonly involve addition and subtraction, fall into three types: **change**, **combine and compare**.

- **The change type**: These situations involve an increase or decrease of some quantity. For example, in the beginning there were ten girls playing throw-ball. Now only four are left. How many went home? This situation is a 'decrease' situation and the operation involved is subtraction. Note that 'decrease' may not always involve subtraction. We can change the question to an addition question for the same situation by asking, 'some girls were playing throw ball. Six of them went home. Now four are left. How many girls were there in the beginning?' Of course, this question may be more difficult for children than the earlier question.

- **The combine type** of situations involve two sets, which are physically combined or which are combined in imagination. For example, in the story of 'Monkeys and the cap seller', 5 monkeys were wearing a cap each and 4 more caps were lying on the ground. How many caps are there in all? In this problem we combine the two sets in our mind and find the number in the resultant set. This is again an addition problem.

  Can you think how from the same situation we can make a subtraction problem?

- **The comparison type** of problems are more difficult for children because the language of numerical comparison is not commonly used in daily conversation. An example, Seema is 6 years old and her sister is 9 years old. How much older than Seema is her sister? The comparison problem becomes slightly easier if we change its wording from 'how much' to 'equality problem'. For example, if Seema has 5 dresses and her sister has 8 dresses, then how many more dresses should Seema get so that both of them have equal number of dresses?

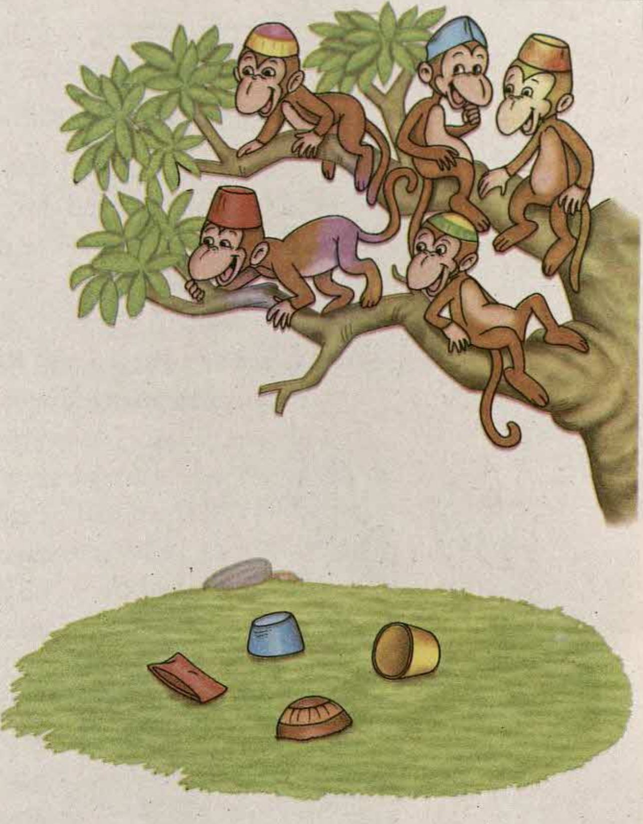


Fig. 2.8

The comparison problems above involve subtraction, but we can make a comparison problem, which involves addition. For example, Shama has 8 bangles. Rajni has 4 bangles more than Shama. How many bangles does Rajni have? Here we need to add 8 and 4 to get the answer.

Think of more situations involving addition and subtraction that are interesting to children.

____

____

STRATEGIES THAT CHILDREN USE FOR ADDITION

Veena, who teaches Class I children, gave a problem to the whole class. Children used different methods for solving the problem Let's have a look at the different ways in which children found the answer.

Problem: Parvez had 4 kittens. His friend Riya had 3 kittens Whenever Parvez and Riya play together their kittens also join them. How many kittens are playing when Parvez and Riya are playing together?

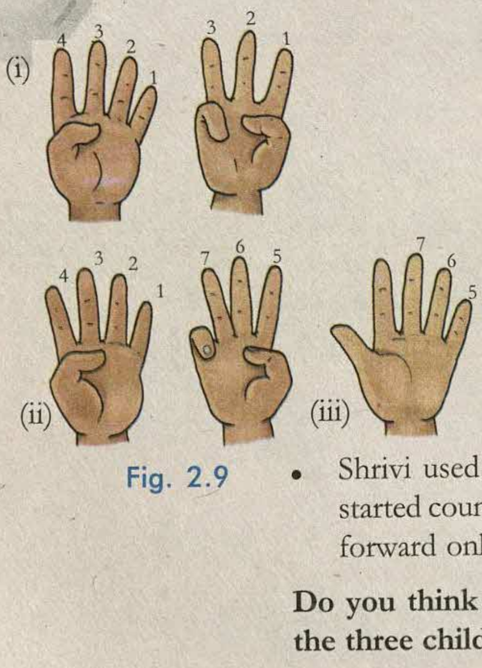


Fig. 2.9

Here are some of the children's responses:

- Prajit counted 4 fingers on his left hand saying "1, 2, 3, 4" and counted 3 fingers on his right saying "1, 2, 3". Then he counted all the fingers together one by one 1, 2, 3, 4, 5,6, 7.
- Naaz also counted 4 on her left hand saying "1, 2, 3, 4". Then she started counting with the fingers on her right hand. But instead of starting from 5 '5, 6, 7'. She said there are 7 kittens in all
- Shrivi used only one hand. She kept '4' in her mind an started counting up from the next number i.e. 5 and counted forward only three numbers, '5, 6, 7.'

Do you think there is a significant difference in the wa the three children are counting?

____

____

The strategy used by Prajit is called 'count all', and the strateg used by Naaz and Shrivi is called 'count on' or 'count up'.

Naaz and Shrivi both are 'counting on' or 'counting up' fro 4 by saying 5, 6, and 7. Notice they have to keep track of when stop the counting. So while saying "5, 6, 7", they are also keeping track in their mind that they have to count forward only three numbers.

Children usually move from the 'count all' strategy to the more efficient 'count up' strategy as they gain experience and become more confident and skilled at counting. It is not desirable that teachers push the children to move to better strategies. The children can learn these strategies gradually, either on their own or by interaction with their peers (classmates and friends) or with the teacher. Sometimes children who have already learnt to use the more efficient 'count up' strategy may fall back to the 'count all' strategy. But there is no need for the teacher to 'correct' the children. Sometimes, when the numbers are too big to be represented on the fingers, children may use a 'count all' strategy, and also may need to use support from objects, icons or tally marks. If children are sufficiently motivated and have interesting situations in which small numbers are to be added, then in due course of time they will move to better and more efficient strategies.

Note for Teachers

Teachers, Please don't use these terms like 'count all' or 'count on' with children, do not push the child to adopt the efficient strategy. We have made an attempt to differentiate between count all and count up, so that teacher understands that there are different strategies.

When we need to add 2 + 7, then it is more efficient to count up two numbers starting from 7 (i.e. start from 7, count 8, 9), than to count up seven numbers starting from 2 (i.e. start from 2, count 3, 4, 5, 6, 7, 8, 9). This modified 'count up' strategy is called 'count up from the larger number'. Some children discover this strategy of their own and move from the 'count up' to the 'count up from the larger number' strategy. Such children are implicitly making use of the commutative property (2 + 7 = 7 + 2).

As children grow older, they also use other more complex and efficient strategies, as the following examples show.

- A child Priya, simply said 4 and 3 is seven. She has memorized the number combination and retrieved it from memory. This strategy is called the **'Known fact'** strategy.
- Anandi said 3 and 3 is 6 and 1 more is 7. She used the strategy called **'Double plus one'**.

Some other strategies which children use for addition are

- **bridging through tens**—For example, to solve the addition problem 8 + 5, the child says 8 + 2 = 10, now 10 and 3 more makes 13.

- **Compensation**—A child was adding 7 to 9. She said, if I take one away from nine, that's eight. Then I give that one to seven and that also makes eight. Eight and eight are sixteen.

In the strategies like 'bridging through tens' or 'compensation', children use their understanding of the decomposition and composition of a number.

The best time to talk about these strategies explicitly is when they come up naturally as part of a discussion. These should not be forced. We, as teachers can encourage children to verbalise their procedures in class discussion, so that other children get a chance to develop and refine their strategies as they hear their peers. This is also true of patterns like the commutative property. Some children realise it on their own as they learn strategies like 'count up from the larger number' then children may learn the patterns and by listening to or observing their friends. By encouraging children to discuss these patterns, they can learn from each other.

STRATEGIES THAT CHILDREN USE FOR SUBTRACTION

Like in the case of addition, children use different strategies for subtraction. A common strategy is to represent the situation using objects, icons or tally marks and to strike out or 'take away' some of them. The easiest representation of subtraction problems are the 'take away' problems. Other problems such as the following can be more difficult to represent and solve.

Rajiya invited 9 of her friends to her birthday party. Six friends came for the party. How many did not come?

Fig. 2.10

- Here one child choses to draw 9 tally marks and strikes out 6, to find out how many children did not come for the party.
- Another child chose to strike out 3 and get the 6 who came to the party. This is also an acceptable and meaningful strategy in this context.

- One child used finger counting to solve the problem. She counted up starting from 6: "7, 8, 9" to obtain the answer '3'. In this strategy child is doing 'counting up'.

  For simple subtraction problems, children usually find the strategy of 'counting up' easier. Here the strategy also fits the situation since the fingers held up one by one stand for the children who were absent.

  Some children use other strategies as well, like

- Count down or backward counting: Children count down in this strategy. For example in this problem they count down from 9 till they reach 6.

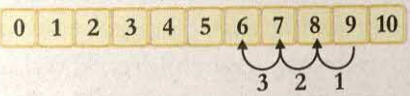


Fig. 2.11

- In this problem, the child may also count down by six numbers starting from 9.

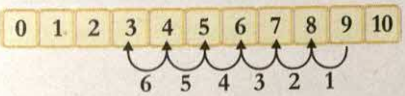


Fig. 2.12

This strategy of backward counting is more difficult to carry out and there are more chances of error. Also, in the backward counting strategy, children are often in a dilemma about whether to start the backward counting from 9 or from 8. These uncertainties make the backward counting strategy difficult and error prone. One can use a number strip as shown in figure to help children in using this strategy. Moving a finger from one number to the other resolves this confusion.

Children also use some other strategies for subtracting small numbers.

- For the birthday party problem presented above, one child retrieved the solution directly from memory and said 9 minus 6 is 3.

  Do you remember the name of this kind of strategy?

- Another child said that 6 + 3 is 9 so, 9 – 6 is 3. He is using the **'additive known fact'** strategy.
- Another child said that 9 and 1 more is 10. Now 10 – 6 is 4, so 9 – 6 is 3. She is using the **'subtracting through 10'** strategy.

The last two strategies use known facts but involve som additional reasoning. So they are called **'derived fact'** strategies

For children who are not confident about finger countin or mental counting, it may be preferable to use tally marks o more concrete representations. Even for children who kno counting strategies to solve subtraction problems, som situations may be more difficult. For example, in compariso or change situations, it may not be obvious to children ho they should model the problem to use counting strategies lik 'counting up'. They may feel more comfortable using concret representations for such situations.

As with addition, children develop these strategies on thei own and it is not desirable that teachers push the children t use more efficient strategies. But children should be given ampl opportunities to apply these strategies. They might make us of concrete material initially and then move on to using icons pictures or fingers, before they us 'known facts' and 'derived facts' with understanding.

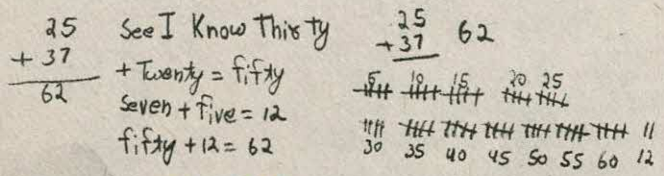


Fig. 2.13

Figure 2.13 shows the differen strategies used by Class I children to compute 12 + 37. Explain what each child is doing.

____

____

ADDITION AND SUBTRACTION UP TO 20

Addition facts

As children become more familiar with addition they follo various strategies to do addition up to 20. Children can be give experiences related to addition in stages:

- Beginning with addition facts till 5 such as 2 + 3 = 5.
- They can then move on to working with facts up to 10 lik 5 + 4, 6 + 4 and so on.
- Further work can be done on double facts up to 20 lik 5 + 5, 6 + 6, 9 + 9 and so on.
- Then they can move on to teen number sentences lik 10 + 4 = 14 (fourteen) , 10 + 9 = 19(nineteen) and so o

Grouping by 5

For addition and subtraction up to 20, grouping by 5 plays a very important role. Children use grouping by five to get the answer quickly.

For example, while adding two numbers like 6 and 7, the child can break up 6 into 5 and 1, and 7 into 5 and 2. She then says 5 and 5 is 10, now 1 and 2 is 3. So the answer is 13.

Grouping by 5 is also helpful for larger numbers. Figure 2.14 shows a strategy to mentally add 66 and 57.

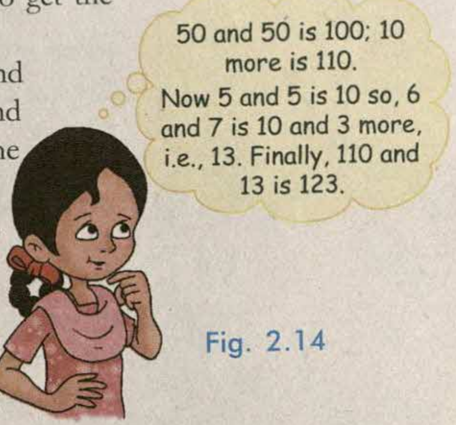

50 and 50 is 100; 10 more is 110.
Now 5 and 5 is 10 so, 6 and 7 is 10 and 3 more, i.e., 13. Finally, 110 and 13 is 123.

Fig. 2.14

The Counting Rack

The Counting Rack on the slate-cum-working board of learning kit helps children to make a mental representation for grouping by five. It contains two rows placed one above the other, with 10 beads in each row. Five beads in each row are yellow and the remaining five are red.

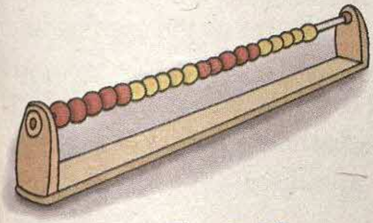


Fig. 2.15

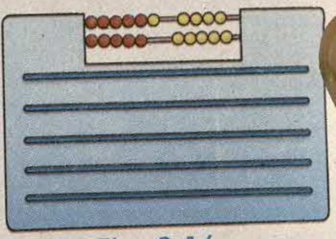


Fig. 2.16

Initially we can give children lots of practice with addition till 5 and then addition of doubles till 5. After sufficient practice as the child moves up to numbers from 5 to 10, **the Counting Rack** becomes useful. To add 6 + 5, the child can do the following:(5 and 5 (all red beads) is 10 and one more (yellow bead) is 11.

Slowly children can be made to move from **the Counting Rack** with beads to the built-in **Counting Rack**, which is part of their bodies. When they are adding two numbers, they can use the toes and fingers on the left side to show one number and the toes and fingers on the right side to show the second number. This is similar to **the Counting Rack** and involves a grouping by 5 – five fingers on each hand, and five toes on each foot. This is a **'Counting Rack'** that they will always carry with them.

Fig. 2.17

STANDARD ALGORITHMS

```
 1
 57      51
 24     -19
 81      32
```

Algorithm is a word used to denote a step-by-step procedure that gives the solution to a problem. By standard algorithm we mean standard procedures that children are taught in schools for solving sums on addition and subtraction, for example, as shown in figure.

These standard methods for solving are widely taught because they are efficient and also general in scope. They can be used for all whole numbers, and can be easily extended to the decimal fractions. However there are some dangers in focusing the whole teaching only on standard algorithm. Because these are the only methods used in the classroom, children begin to think that there exists no other correct method of doing the basic operations. Also the standard algorithms require the child to deal with the digits in the number separately. For example, in the addition problem above, 7 and 4 are to be added first and then 5 and 2. This suppresses the wholeness of the number and the fact that 57 is 50 and 7 and not 5 and 7.

The algorithm is very different from the way the child thinks about numbers and the way in which she makes use of intuitive computational procedures or strategies discussed in the previous sections.

The standard algorithm requires a series of rules to be learnt and many times children don't appreciate how the rules work. They apply rules in a mechanical way and in the process make errors.

Look at the errors made by Suman:

Try and find out the (wrong) rules that Suman has applied.

```
(a)   26
    +  7
     213

(b)   53
    - 14
      41
```

What is important for us as teachers is to introduce the standard algorithm at the right time. Before introducing the standard algorithms, we need to let children develop their own intuitive strategies to solve problems. Even after teaching the algorithms, it is important to regularly practice solving problems mentally, so that children develop a rich set of informal procedures and strategies. And when we introduce the standard algorithm, we should carefully explain the steps and relate it to other concepts such as place-value. Teaching the standard algorithm without helping the child to get a grasp of the underlying concept will lead to inadequate understanding of the procedure. Good understanding of the algorithm will help in better retention of the rules of the algorithm and allow the children to appreciate the reasons that underlie the rules.

For teaching written algorithms, there is a need to link concrete procedures and the written algorithm. Here is an example where the teacher used bundles of 10 sticks and loose sticks to introduce the algorithm of addition. Before this, she had already done enough work with children on representing number in tens and ones using sticks bundles of 10 and loose sticks (as discussed in section on place-value). Using this representation, 34 can be represented as shown below:

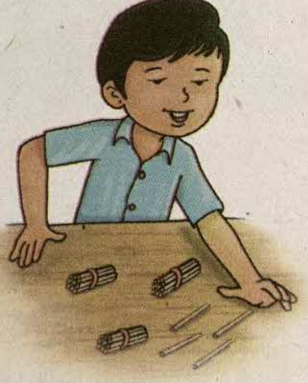


Fig. 2.18

Suppose we want to add 34 and 26.

Try to do this addition using stick bundles. First represent both 34 and 26 using tens and ones. Then do the addition first of the ones and then of the tens.

Study what happens. What is the meaning of 'carryover' when we do addition with bundles and loose sticks?

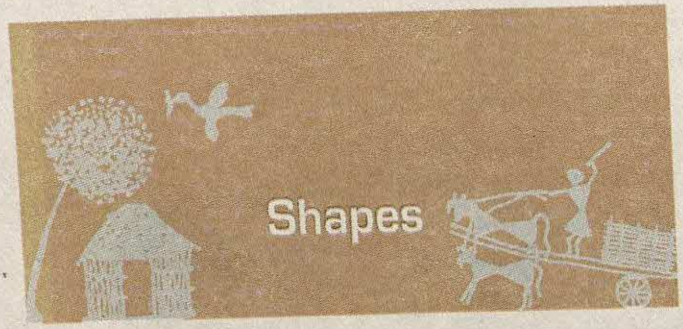
____

____

____

In the mathematics classroom different forms of representation and usage of different kinds of material for the same concept need to be encouraged. For example, along with sticks, loose beads and necklaces, blocks and play money notes of 10 and 1 given in the learning kit can also be used in a similar manner.

In Classes I and II, children learn many informal strategies in dealing with addition and subtraction of small numbers. They need to be encouraged to develop such informal strategies since it helps to build a 'number sense'. They learn to interpret, represent and solve simple problems. They are also introduced to the standard algorithm with sufficient connections being made to other important concepts such as place-value.



Shapes

3

In Classes I and II, a child begins her journey in geometry by exploring and understanding shapes more deeply. From everyday experience she already has some understanding of shapes. She has seen and played with many different objects. She has touched them, felt their texture, observed their structure and experienced some of their properties. She may have played with a ball, a bottle, a box, with pebbles, vegetables, seeds, etc. She may have thrown them about or thrown them along the floor. She may have observed some interesting differences.

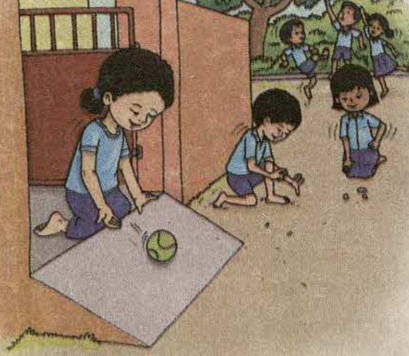


Fig. 3.1

1. The ball rolls in all directions.
2. The bottle rolls in only one direction.
3. The box does not roll, but only slides when we push it along the floor.
4. Rolling objects go further than sliding objects and so on.

Thus the child has made important connections between the shape of an object and its properties, in this case, how the objects move in different situations. She may have even formed her own conclusions and generalisations about these relations. She may be able to explain, "why a box cannot be rolled" in her own words. Similarly, when she pretends that a box is a house, she is noticing the similarity in the shape of the box and a house.

Fig. 3.2

Thus, the phenomenon of shapes is not new to a child. She has already enough knowledge that can form the foundation for further understanding. The child may also know some words related to shape: round, flat, narrow, pointed, sharp, fat, thin, etc. She can also use the names of objects to denote shape concepts, for example, when she says "*ye gaind jaisa hai*" or "*ye dibba jaisa hai*", etc.

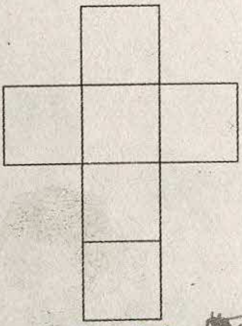


Fig. 3.3

The teaching and learning of geometry in Classes I and II revolves around the exploration of basic three-dimensional (3-D) and two-dimensional (2-D) shapes of objects found in the environment. The child explores and begins to understand the difference between (2-D) and (3-D) shapes. In Class III and beyond, the elements of shapes such as sides, vertices and angles are explored – the foundation for the exploration of elements of shapes is laid. The children also explore complex shapes and repeating patterns that can be made with shapes (see the chapter on 'Patterns'). In Classes III and IV, they explore tiling patterns more systematically. In Class IV and beyond children explore the relation between 2-D and 3-D shapes through exploration with nets (two dimensional cut-outs from which three dimensional shapes such as cube, cuboid, cylinder, cone, etc. can be made).

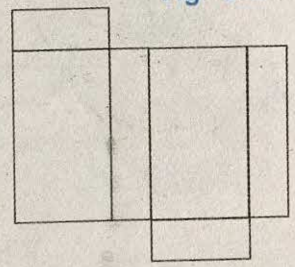


Fig. 3.4

The major learning outcomes of teaching shapes in Classes I and II are to

1. describe the shapes of various familiar objects in her own vocabulary;
2. identify various elements of an object such as edges, corners and faces in the course of exploring and playing with them;
3. explore and communicate the association between an object (3-D) and its shape (2-D);
4. draw simple shapes on paper; and trace and explore the outlines of objects on paper.

In the classroom as teachers, we sometimes forget that the child has some previous experience and intuitive understanding of objects and their shapes. Often we emphasise the formal names for shapes without making the necessary connection to

daily life objects or the children's intuitive understanding. Here is a small snippet from a classroom.

Teacher: (Pointing to a tennis ball) What is this?

Child: Ball.

Teacher: What is the shape of the ball?

Child: Round

Teacher: Round or circle?

Child: (confused)

Teacher: What is this according to you? Round or circle or circular or sphere or spherical?

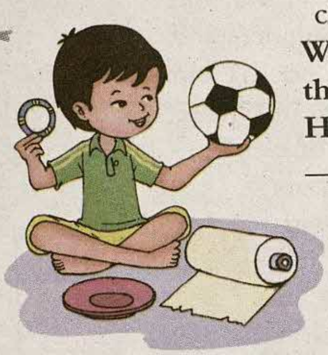


Fig. 3.5

What is the teacher trying to do here? Do you think the child is deepening her understanding? How would you carry on this discussion?

____

____

Since children are familiar with the shapes of ordinary objects, it is better to explain differences between shapes by making a connection with other objects—this is round like a ball, it is round like a plate, or like a bangle, or like a roll of paper, etc.

When children use their own language or common vocabulary, they can communicate what they find through their explorations. It helps them generalise and to understand the concept better. Later on they can relate this base of understanding to the formal mathematical vocabulary. (Also see the chapter on 'Language and mathematics'.)

Teacher may use the formal vocabulary herself while describing shapes; she need not expect the children to learn this vocabulary. The children should be encouraged to freely use the language of daily life while describing and naming shapes.

EXPLORING SHAPES

The school and the child's environment offer many opportunities to explore various objects and their shapes. The teacher needs to create situations in the classroom for such exploration. For children, exploring shapes can be fun. The teacher can create an activity where children explore and share their observations with other children.

Snippet 2

A teacher of Class II had children explore shapes sitting in a circle. She had collected several objects for this activity—geometry box, book, water bottle, pencil, scale, chalk, lunch box, etc. She picked up one object and passed it around. Each child held the object for half a minute and said something about its shape or feel. She or he had to try to say something new that was not said by another child. When one object had completed a full 'round', a new object was introduced.

Rashid: The geometry box is smooth and cool.

He said it by holding the box against the cheek.

Geysar: When I put the edge against the skin it is hard.

The teacher asked if the box had any corners. The children noticed that the corners of the geometry box were rounded. Similarly, they saw and felt the corners of the book with their fingers. In this activity children noticed important elements of the shape–face, edge and corner. Keeping a systematic record of the observations discussed could extend this activity further in a more meaningful way. It will help in generalising the properties of shapes.

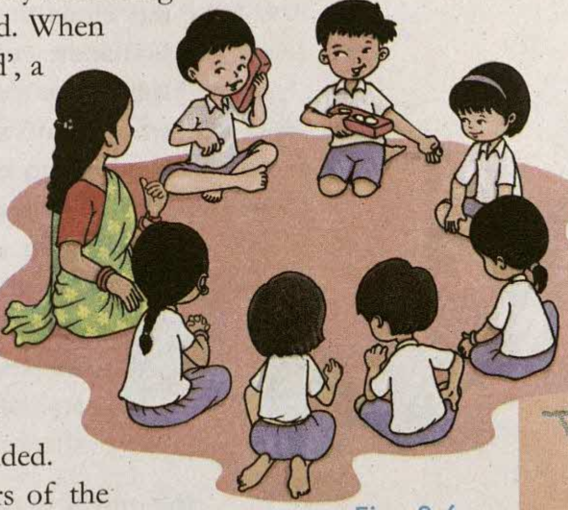
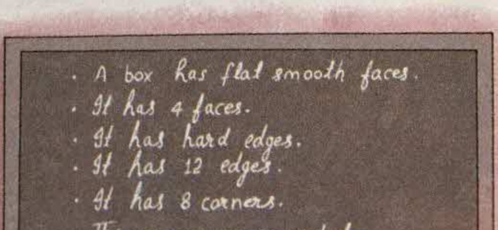


Fig. 3.6

The teacher may write down some of the observations and discussion points of children on the blackboard. This will help the children notice the elements of shapes better.

Another teacher of Class II made her children explore shape through a slightly different game. She made her children sit in the circle and passed various objects like pen, die, duster, bangle, etc. Each child held the object and demonstrated that object in a novel form. For instance, when the pen was passed, one child



· A box has flat smooth faces.
· It has 4 faces.
· It has hard edges.
· It has 12 edges.
· It has 8 corners.
· The corners are rounded.

Fig. 3.7

demonstrated the pen as lipstick, the other showed it as a thermometer. Similarly, when an eraser was passed, one child pretended that it was a soap bar and another that it was chewing gum.

How do you think this activity is useful for shapes?

____

____

After the 'circle time' was over, children played with the models of shapes: cube, cuboids, cylinder and cone. The learning kit has a set containing models of shapes, which are covered with velcro. Velcro on the models allow the children to stick one shape to the other on any face, edge or corner. This allows for a lot of creative joining and mixing and making more complex shapes. In making different objects through these models children further learn about the various elements of different shapes, they also learn about the similarities and differences in the elements of different shapes.

Fig. 3.8

Fig. 3.9

Snippet 3

Child 1: Let's make a table.

Child 2: (picks up the cuboid and holds it as the base of the table.)

Child 1: Now we need to make it stand, so lets add four *belans* (cylinders)

Child 2: Instead of these, we can place two *dibbis* (cuboids) like these.

One child made the table with a disc as a base and cylinder as a stand. Both the children are not just applying their creativity but are also learning about the basic properties of different shapes.

In all these activities children were taking an important step – analysing a shape into its elements. In geometry, shapes are defined in terms of the relation between the elements, not in terms of how the shapes appear to the eye. Many children think the square and diamond (that is, a square rotated so that its diagonal is vertical) are different shapes because they *look* different.

Geometrically however, we think of them as the same shape – a square, because their sides are equal and each of their angles is a right angle. The square shape is defined in terms of the sides and the angles, and not in terms of how it appears to the eye. This is an idea that children take time to learn – learning that shapes are based on the relation of elements continues through all the primary years. We can help children in understanding this meaning of shape, by helping them learn about the elements of a shape such as sides, vertices (corners). The activity of exploring shapes is a good way to get them to notice such elements of shapes.

The NCERT Class II mathematics textbook contains an interesting activity, where a blindfolded person is feeling inside a bag and describing a shape, while children guess the shape or object inside the bag (also, see below). This activity as well as the suggestions for more activities are given below. These and the activities described above help in building children's vocabulary about shapes, since they have to communicate what they observe to others.

Activity: **Feel and describe**

A blindfolded child touches and feels an object kept inside a bag. Nobody can see the object. This child who is feeling it describes the shape and texture of the object to the whole class. The children in the class try to guess what the object is. The teacher can write their guesses on the board. After a few minutes the blindfold is untied and the object is taken out. Children see it and check whether their guesses were right and discuss whether the description given by the blindfolded child was clear, how it could be made better, etc.

Activity: **Bol Bhai Bol**

Class is divided into two groups. Both the groups play this activity in a singsong manner. The first group first sings "*kya hai lamba kya hai gol, bol bhai bol jaldi bol*". The second group responds "*balla lamba, gaind hai gol, bol bhai bol, bol bhai bol.*" Now the second group asks from the first group, "*kya hai lamba kya hai gol, bol bhai bol jaldi bol*". The first group responds "*pencil lambi, ladoo hai gol, bol bhai bol, bol bhai bol.*"

In this way the game goes on and children classify various objects according to their properties.

RELATING THREE DIMENSIONAL AND TWO DIMENSIONAL SHAPES

One of the problems of learning geometry is to distinguish between three-dimensional and two-dimensional shapes and understand their relationship. Common objects in the world are three dimensional, and hence these shapes are familiar to children. Shapes drawn on paper are two-dimensional. Pictures, are also two-dimensional shapes, but it is difficult to see them as two dimensional. This is because the artist uses various techniques to create the illusion that they are three-dimensional. Techniques to draw three dimensional shapes on paper require skill and tranining.

In Classes I and II children explore and understand the basic difference between three and two-dimensional shapes. You can put things inside a box, but a box drawn on paper is flat and you cannot keep anything inside it. They also explore the relation between 3-D and 2-D shapes by tracing the outline of objects on paper, which is one way of reducing a 3-D shape to two dimensions.

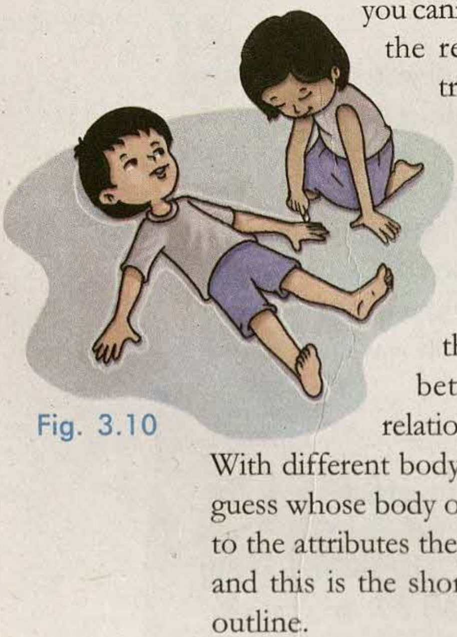


Fig. 3.10

Activity: ***Trace your friend's body***

In this activity one child lies down on the floor and the other traces her body. With this activity children get to see the relationship between the solid body and its outline, i.e. relationship between a 3-D shape and 2-D shape. With different body outlines, teacher can have discussions like, guess whose body outline is which. Children respond according to the attributes they observe like, Rekha is shortest in the class and this is the shortest outline, so this must be Rekha's body outline.

NCERT mathematics textbook Class II has a chapter on 'Footprints'. In this chapter children are expected to match the animals with their respective footprints. Looking at the shape of the animal's foot, the child needs to match its shape with the footprints on the ground. Similarly they trace and compare their own footprints with their friend's footprint.

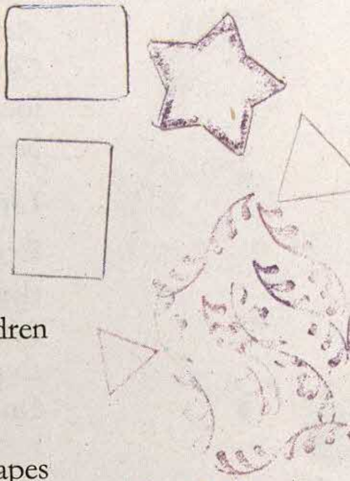


Fig. 3.11

Sunita, a teacher in Class II made her children do the tracing of hands. Children did the tracing and guessed which trace is of which child. Following this, Sunita asked them to make *mehandi* designs on the outlined hands. Children enjoyed the activity a lot.

Activity: **Tracing from stamping containers**

The learning kit has stamping containers in regular, 3-D shapes like cube, cuboid, cylinder, which can be used for tracing and stamping. Other shapes like 5 pointed star, leaf, etc., can also be used to make interesting patterns using 2-D shapes.

There are two main objectives that can be achieved by doing all these tracing activities. First they help the child observe the relationship between a 3-D shape and its associated 2-D shape. The second objective is to help the child in identifying the similarities and differences between traces of different objects. One object can give traces of two different types, for example, the trace of the base of a bottle is different that of its side. Even the traces of two bases can be different. Also traces of two different objects can be similar. For example, trace of a bottle cap is similar to the trace of a coin. When children start exploring the traces they encounter interesting shapes like circle, square, triangle etc.

While exploring 3-D shapes they simultaneously also explore the elements, which are two dimensional, namely, faces. (3-D objects also have elements that are one dimensional, namely, edges, and elements that are zero dimensional, namely, vertices. However one dimension and zero dimension are concepts which are not easy to understand and may be dealt with in higher classes.) In higher classes, using paper cut-outs (called 'nets') to make boxes, cubes and other shapes is a good way of relating 2-D and 3-D shapes.

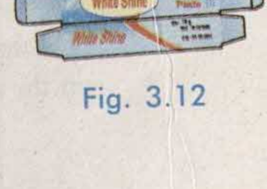


Fig. 3.12

Activity: ***Exploring how a box is made***

Children cut open an empty box made up of thick paper, e.g. toothpaste carton, chalk box, etc. They observe and discuss how the box is made out of a plane sheet of thick paper.

Lines

Explorations of lines provides the foundation to explore the elements of shapes like sides, vertices and angles. Children at this stage learn about straight and curved lines. They also learn about three types of lines, standing, sleeping and slanting lines. One of the main objective in Class II is to help children explore and identify the lines which are all around them and to draw different types of lines.

Exploring Lines

NCERT textbook for Class II uses an interesting way to help children look at various types of lines. Many stick-drawings and pictures of digital clocks, folk art, traffic police signals, etc,. are shown. Children explore and identify the types of lines that have been used in these drawings.

Drawing lines

The use of ruler to draw straight lines of specific measurement can be avoided in Class II as most children find the rulers difficult to handle. At this level children can be given space to draw free hand lines and to use various other techniques to draw lines. Some of them are given below.

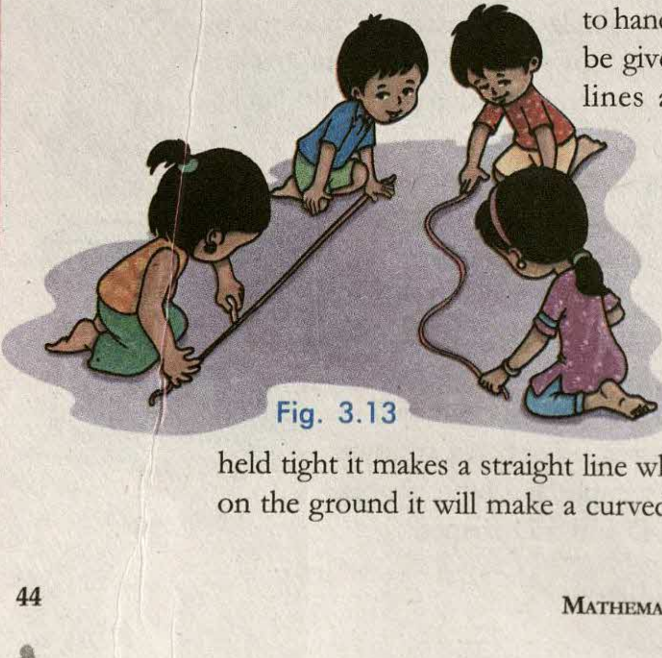


Fig. 3.13

Activity: ***Make a line with string***

Children can make lines in the ground using the string. When a string is held tight it makes a straight line whereas, if you let it lie loose on the ground it will make a curved line.

Activity: ***Make a line by tracing***

Children can use the straight edges of various objects like pencil box, book, stick etc. to draw a straight line.

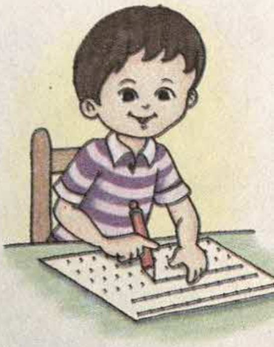


Fig. 3.14

Activity: ***Join the dots***

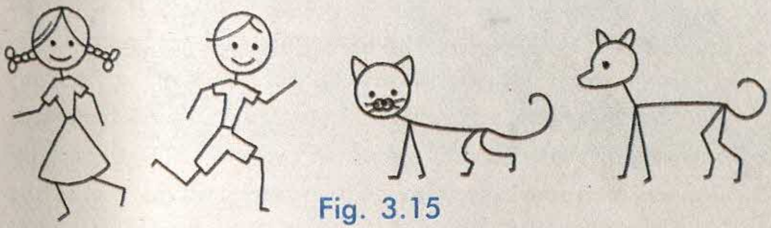
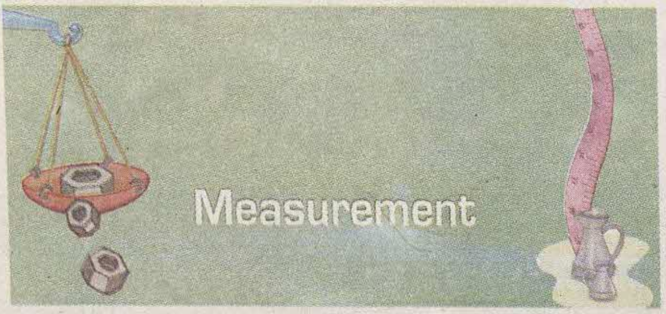


Fig. 3.15

A dot grid can be given to the children to join the dots to make lines.

Children have fun in making stick-drawings using the sticks followed by making stick-drawings on the paper.

In this way teacher can provide situations to help the child explore lines, which will further form the basis for making progress in geometry.



Measurement

4

Fig. 4.1

In everyday life, we come across many situations where we deal with quantities, for example, while buying provisions or cooking a meal for guests. We also use informal measurement and estimation—estimating the time required to fill a bucket or to boil milk, buying cloth, budgeting, setting the volume of the television and so on. Making decisions such as which two wheeler to buy may depend on understanding mileage, price, running cost, etc. Measurement is essential in occupations such as tailoring, carpentry, building and so on. It is fundamental to science and hence understanding and using measurement is important for learning science. Understanding issues related to the environment such as pollution, weather, rainfall, etc., require the understanding of measurement of many physical quantities. Understanding and having control over health and illness may require some knowledge of measurement of body temperature, doses for medicines, understanding what is meant by blood pressure or blood sugar level, etc. In general, knowledge of measurement and familiarity with the different contexts of measurement is important for functioning effectively in daily life or as a citizen.

Fig. 4.2

Children are often involved in activities like dressing dolls, comparing whose pencil is longer or whose shorts have more pockets, building towers and comparing their heights, checking whether there are enough sips of water left in their water bottles, how much time is left for the lunch break and so on. In this way children acquire an informal understanding of several physical attributes such as length, weight, volume and time. We can use this experiential learning of children and these natural contexts as a basis to develop and plan learning activities. It is preferable that the activities have an inbuilt challenge in the form of a problem which children can easily relate to and find interesting. We should encourage children to explore and reflect on the given problem situation and provide enough opportunities to develop the natural understanding related to the concept of measurement.

Fig. 4.3

Children learn about the attributes of measurement such as length, weight, volume, money and time in the primary classes. They may also be introduced to other attributes such as area. They learn first to clearly recognise the attribute that needs to be measured and distinguish it from other attributes. For example, they need to distinguish weight from volume, although there is a relation between weight and volume. They also need to understand that the same attribute may take different forms. For example, measuring how long a pencil is or how big a tree trunk is or what the distance from the school to the home is, are all measurements of the attribute of length. Children slowly and with increasing practice in measurement come to understand these various aspects.

Children begin in Classes I and II with the direct comparison of attributes that can be easily perceived such as length or weight. While making comparisons, they learn the vocabulary of comparison. Sometimes the attributes of objects can be conveyed directly, for example, the length of pencils. But if we want to compare whether the length of the blackboard is more than the height, we may need to use another intermediary object like a string. Children move from direct comparison to indirect comparison using an intermediary object, and then to measuring with informal or non-standard units such as hand spans, footsteps, cups, bottles, sticks, etc. There is a big difference between comparison and measurement, because measurement involves the use of number. The number says how much there is of an attribute (say, length). We get the number by the repeated use of a chosen unit to 'cover' the attribute. For example, to measure the length of a pencil, we may choose the matchstick as our informal unit, and observe that five matchsticks 'cover' the length of the pencil. So the pencil is '5 matchsticks' long. Similarly we may say how many cups of water are needed to fill a bottle, so the volume of the bottle is so many cups. Children need to understand the use of the counting numbers in this new context of measurement, which involves counting how many times the unit 'covers' or 'fills' the measured quantity.

When children use a scale to measure length, they simply read the measure from the scale. So it is not obvious that the unit has been repeatedly used to measure the length. Many children do not understand why we use a scale or how it is constructed.

It is preferable to move to standard units only after children have understood why a unit is necessary in measurement, and how the unit is repeatedly used to measure the attribute to obtain a number or 'measure' for the attribute.

Fig. 4.4

They can appreciate this better when they have chosen or constructed their own unit in the course of solving a problem requiring measurement. Even after standard units are introduced, it is important to stress the idea that the unit is used repeatedly to cover the attribute. So it is better to introduce measurement by a scale only after children have used informal units. Moving too quickly to measuring by reading from a scale can result in an incomplete and partial understanding of measurement.

In Classes I and II, the focus is only on understanding the attribute being measured, developing the language of comparison, and on measurement using non-standard units. Moreover we only focus on simple attributes that can be perceived directly such as length, weight and volume. Afterwards, in Classes III to V other measuring activities will be introduced which involve exploration of more attributes, and working with standard measurement units (Kg/gm; cm/m/km; l/ml; sec/

min/hrs) and measuring instruments (scale, measuring tape, meter stick, marked beakers, clock, weighing balance, etc.). Children will understand the different units used commonly and also get a feel for how much quantity these units specify.

In Class III and beyond children solve problems involving standard units of length, weight, volume, time. They also have to relate the larger and smaller units commonly used to measure the same attribute such as metre and centimetre and also convert one to the other. They learn to go beyond whole numbers and apply simple fractions to quantities. They learn about attributes such as perimeter and area and extend their understanding of volume of liquid in a container to include the volume of a solid body, at first intuitively and then by informal measurement. For all this to take place, it is important that their foundation in the concept of measurement becomes strong.

Comparing and ordering of objects according to a certain attribute such as length, volume or weight are the initial activities that lay the foundation for measurement. While comparing or measuring, an effective way of developing a feel for measurement is to encourage children to make a guess or a visual estimate and then verify this by carrying out a more systematic comparison or measurement. There should be scope for discussion in the classroom where strategies or reasoning used by different children can be shared and discussed. Discussing a strategy helps to discover other strategies or to generalise the strategy. It also helps the teacher gain insight into the level of understanding achieved by children.

Fig. 4.5

LENGTH

Children are growing all the time. They are growing out of their clothes and shoes. They are often heard saying, "You will not be able to sit on this toy bicycle as you are bigger than me".

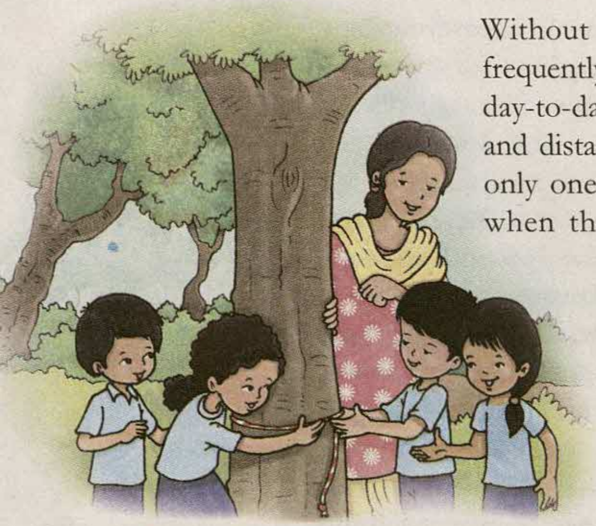


Fig. 4.6

Without being explicitly aware of it, they are frequently involved in comparing lengths in their day-to-day activities. Measuring height, breadth and distance are all actually the measurement of only one physical quantity, namely, length. But when the length is associated with different orientations, or different scales, then different words are used making it difficult for children to perceive that the same attribute is being measured. When we are focusing on length in the vertical upward direction, we call it height, and in the vertical downward direction, we call it depth. When the length is horizontal, we sometimes say length (for the longer length) and sometimes breadth. 'Thickness' refers to the dimension which is usually shorter than and different from length and breadth. When we take a measurement around an object, like the chest, or a tree trunk, we call it 'circumference'. When the length is large in comparison to the dimensions of our bodies, we call it distance. Thus the vocabulary associated with comparison of length includes words like long/short, small/big, thick/thin, tall/short, high/low, shallow/deep, far/near, narrow/wide, etc.

The initial activities centred on the attribute of length involve comparison and ordering. Comparison of length without measurement may be done by observation, by superimposition, or by using an intermediary object. Different situations and problems lead the children to using these different strategies of comparison. For example, to compare the heights of two children, one can observe directly. If the heights are nearly equal, one can refine the observation by using a large hardcover book which can be placed on the heads of the two children standing back to back or facing each other and see which side of the book is higher.

To compare the length of two pencils, or two photographs, one may bring them next to each other or superimpose them. To find out which window is longer, we may use a string as an

intermediary object. To find out which tree trunk is bigger, or whose waist is bigger, we can wrap a string around the trunk or waist to measure the circumference. When we need to communicate the length of an object to another person who is not present, we may need to use a commonly available informal unit. For example, we may say, take a piece of paper that is 4 matchsticks long and 4 matchsticks wide, assuming that matchsticks are commonly available and are of the same length. By posing problems involving these different situations, we can lead children to come up on their own with these various strategies for comparison.

EXAMPLES OF ACTIVITIES FOR CLASSES I AND II

1. Whose arm is the longest—Group activity

Firstly, let children guess whose arm is the longest. Then provide paper strips of equal length to all the children ensuring that the length of the paper strip is around 50 cm. Children will measure the length of their arm using these paper strips and cut off the extra portion of the strip. Let children find out by observing the strips that who has the longest arm and who has the shortest.

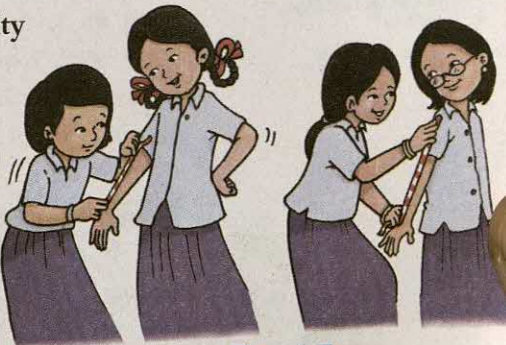


Fig. 4.7

Now children can be asked how they will find out who all have arm of equal length. Children may come up with the idea of comparing strips.

Similarly, we can ask children to find out whose hand span is the longest and whose is the smallest.

2. Comparing lines drawn on paper

When children are shown two pencils of the same size, kept side by side, they can find by direct observation if both the pencils are of the same length. But when two equal lines are drawn on paper at some distance from each other, it becomes difficult for children to directly perceive that they are still of the same length. Using a thread and marking it, children can compare the length of the two lines and find that they are of equal lengths.

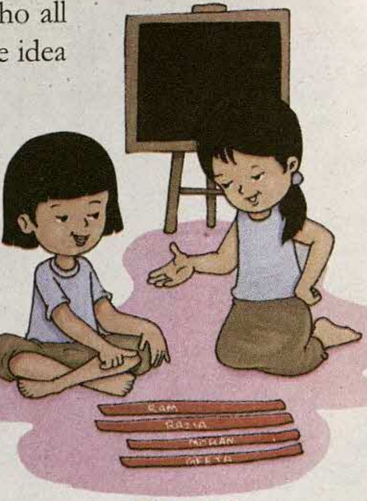


Fig. 4.8

My pencil is shorter than my stick

Fig. 4.9

This is an example of a problem where children need to use a intermediary object to carry out the comparison. Similarly, the can compare the length and breadth of a notebook or the heigh of two plants in two different pots using a string.

3. Comparing with a reference length

Another activity which is interesting to children is to compar lengths against a reference length. We can give the children stick which serves as the reference length. Children can be aske to first guess and then find out which objects in the room ar longer than the stick. Which objects are shorter? What object are of the same length as the stick?

4. In the same way, the length of many things in the classroom such as windows, door, blackboard, table tops, can b compared. Children can use intermediate objects like strin given in the learning kit for such comparison. They can also us informal units like the hand span, footstep, etc.

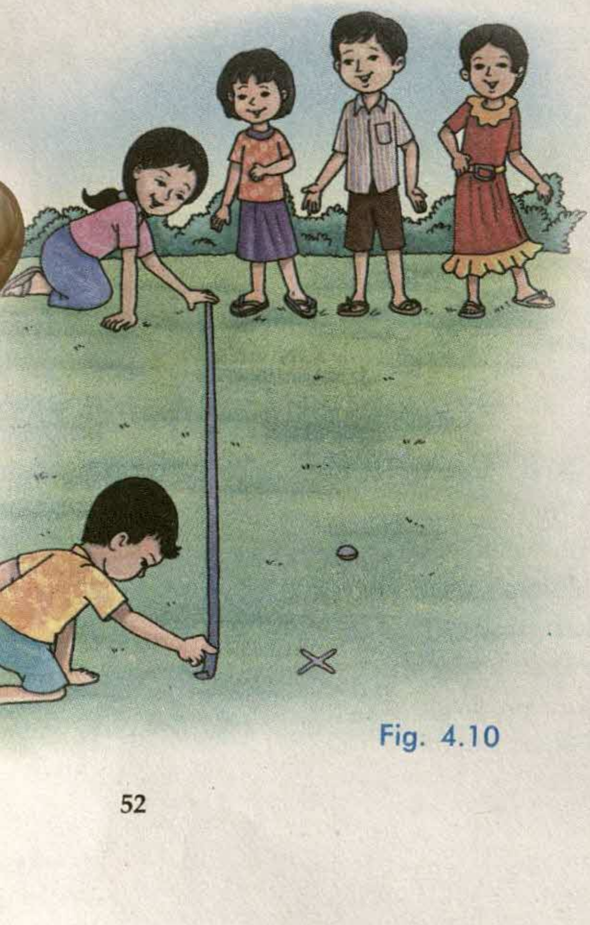


Fig. 4.10

5. Who is the nearest?—A game

A simple but interesting game can be played by marking an 'X' on the ground with chalk. A line is drawn about 10 feet away from the 'X' mark. Children take turns to throw a piece of chalk as close as possible to the 'X'. Each child's throw is marked with a dot and by writing the child's initials next to the dot. At the end, to find the winner, children compare to see whose throw is nearest to 'X'. When two children's throws are almost equally close, one may use the string to decide. Another idea is to use 'footsteps' or 'handspans' to compare the distances.

In all these activities, the children may be encouraged to use the vocabulary of comparison: taller/ shorter, longer/ shorter, nearer/farther, etc. When more than two objects are compared, it becomes meaningful to use the superlative forms: tallest, shortest, longest, etc. Conceptually, comparison of three or more objects and arranging them according to the size of the attribute is more advanced than comparing two objects. Ordering three or more objects according to size is sometimes referred to as 'seriation'.

The language associated with measurement and comparison is developed only by actively using the language. So in these activities it is important to get children to talk and to listen to each other and to the teacher, and not restrict students only to completing worksheets. Oral communication and speech, where children use the vocabulary is of utmost importance. Open ended tasks and problems which invite participation from the students are useful in encouraging them to speak.

Children's spontaneous activity

Here is an example from Class II. When the teacher entered the class, a few children were talking about matching the length of a pencil to other things.

Child-1: My pencil is so long! Can you match its length?
Child-2: Yes, I can put both my pencils to match the length of your pencil. See!
Child-3: I will put my pencil and an eraser to match the length of your pencil. See they are equal to your pencil.

This spontaneous activity is a useful starting point for introducing the idea of covering a given length. From this starting point, children can move on to covering the length with identical units, such as matchsticks, and from there to using a convenient unit based on the body such as the hand span.

Exemplar questions to make worksheets

(1) Worksheets on comparison can be made interesting by giving instructions to colour one out of a pair. For example, 'Colour the longer stick blue and the smaller stick red'.

(2) As mentioned earlier, it is important to develop the language and vocabulary related to measurement among children. We can do this only by having opportunities for children to express themselves orally. Pictures are a good

medium to get children interested and to let them star talking. There is an interesting picture book by the Nation Book Trust called *Tails by Hydrose Aaluwa.* One can use th picture book and can ask questions as following:

- What do you observe? • What do you find in this pictur
- What can you say about the tails of the animals.

VOLUME

Like length, volume can also be directly perceived. But unlike lengt it is usually not easy to compare two volumes by direct observatio Volume refers to the amount of space, and in the primary classe we talk mainly about the volume occupied by a solid or liquid. I Classes I and II, we focus on the easier attribute, which is the volum of liquid in a container. So we focus on the volume of water in bottle, cup, jug, etc. It is only later, in Classes IV and V, that childre learn about the volume of a solid body. Solids neither chang volume nor shape as we handle them. Liquids change shape easil but their volumes do not change as we handle them. This makes easier to measure the volume of liquids since we can pour the liqui out into containers whose volume is known. To measure the volum of solids, we must make indirect measurements using a liquid, fo example, by finding out how much liquid is displaced by the soli when we immerse it in the liquid.

In some textbooks, the word 'capacity' is used instead o 'volume'. The capacity of a container is the total volume of th liquid that it can hold. However it is preferable to use the wor 'volume' so that we can use this word in situations where th container may only be partly full. For example, we may want t compare two buckets which are partly filled with water. In thi situation, if we ask which bucket contains more water, we ar asking for a comparison of the volume of the water in the tw buckets, and not for a comparison of the capacities of the tw buckets. In some situations, it may be more relevant to compar the capacities of the containers. For example, we may want t know which bucket will hold more water when it is full.

Young children can easily make mistakes about which container has more water. The well-known psychologist Jean Piaget conducted a famous experiment, where children observed that the water contained in two identical glasses were equal. Then he poured the water from the first glass into a thin, tall beaker, and the water from the second glass into a wide beaker. Then he asked the children if the two beakers contained equal amounts of water. Young children usually said no, and said that the thin, tall beaker contained more water. This was surprising because the same children had said earlier that the two glasses contained equal amounts of water. Piaget concluded that young children do not 'conserve' volume.

Although we cannot 'teach' children to conserve volume, as they gain more experience in working with liquids and comparing and estimating quantity, they grow more confident in their understanding of volume. In Classes I and II, they are ready for such experiences. So here we focus only on comparing volumes by visual estimating, by pouring out and estimating how many of one container (full of water) is needed to fill another container.

Some of the Activities that can be done are as follows:

1. Take identical glasses and ask children to pour an equal amount of water in each glass. Let them ask other children if each glass contains the same amount of water. Introduce them to the use of the phrase, 'equal amount of water'.

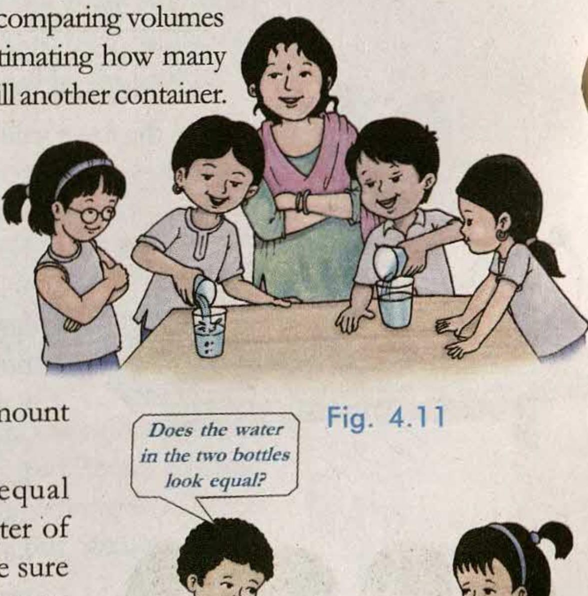


Fig. 4.11

2. Take two identical glasses with equal amounts of water. Now pour the water of each glass into a different bottle. Make sure that the shapes of the two bottles are very different. Ask the children to discuss what happened. Why does the water in the two bottles look unequal? What happens if they pour it back into the glasses?

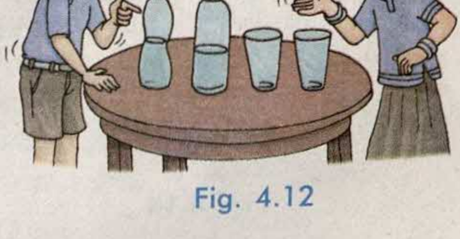

Does the water in the two bottles look equal?

Fig. 4.12

3. Pour some water into two glass bottles of different shapes and ask the children which bottle contains more water. Make sure that the bottles are less than half full, and that the difference between the amounts of water is not much. This makes it more challenging to the children. Ask the children how they can check if their answer is correct. You can keep some identical glasses in front of the children, and prompt them by asking, 'can we use these glasses to find out which bottle has more water?' One of the children may suggest that we can pour out water from the two bottles into two identical glasses and check. Ask the other children to respond to this suggestion before actually trying it out.

   How will the children respond if there was only one glass instead of two?

4. Get bottles and containers of different shapes and pour some water in each of them. Ask the children to guess which has the most water, which has the least. How will they check if they are correct?

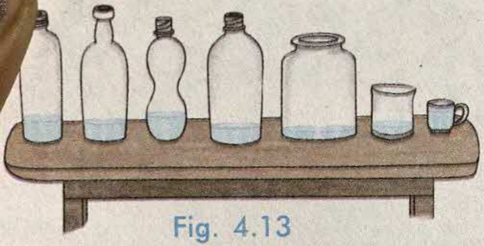


Fig. 4.13

Fig. 4.14

Estimation activities

1. Use bottles of different shapes in Activity 4. Ask the children to make a guess about which bottle will hold the most water and which bottle will hold the least water. Ask them to write down their guesses. Now take a small cup and pour one cupful of water in each bottle. Now let them make a second guess about which bottle will hold the most and which the least amount of water. They can fill up 2 more cupfuls of water in each bottle and make a third guess. When are they confident
2. Take two bottles of different capacities. Ask the children to guess how many cupfuls of water each bottle will hold. Let them write down their guesses. Then ask them to check by actually pouring cupfuls or by filling the

stamping containers from the learning kit with water and pouring it into the bottle.

3. Children can also use the stamping containers provided in the kit for volume estimation. At first they can estimate how many cubical containers are required to fill the cuboidal container, later on they can check their estimates.
4. Exploring volume with the use of stamping containers will also help them to explore which shape in the kit can hold more water and they will be able to correlate it with the attributes of solids. Children could be asked to reason about their estimations as it will help them to think logically.

As one can see, the main aim of the activities dealing with volume is to give children an experiential base which will be helpful in building their understanding of measuring volume. We can also focus on the language of comparing volume and on learning to use words and phrases such as, more water, less water, one cupful, two cupfuls, can contain or hold more, etc. Most of these activities can be done in a small or a large group guided by the teacher.

WEIGHT

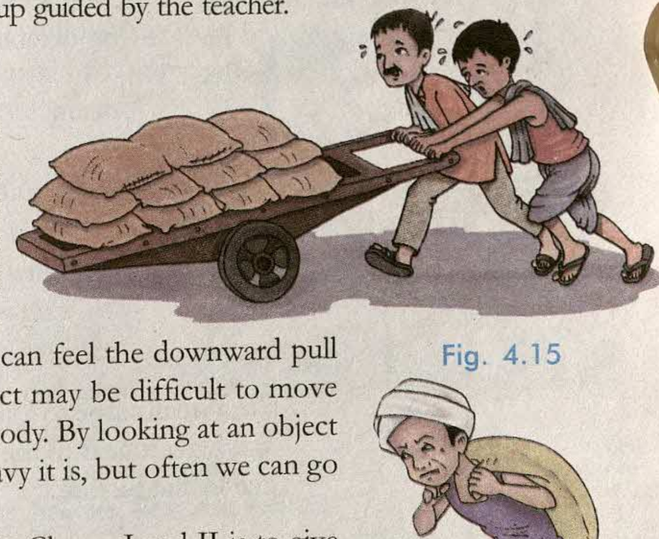


Fig. 4.15

Weight is also an attribute that can be perceived directly by our senses. In this case however we do not perceive it using our visual sense, but by feeling it with our hands and bodies. A heavy object is more difficult to lift. When we lift it, we can feel the downward pull on our hand. An even heavier object may be difficult to move even if we push it with our whole body. By looking at an object we can make a guess about how heavy it is, but often we can go quite wrong in this judgement.

With regard to weight, the aim in Classes I and II is to give children experiences that will help them focus on this attribute. They also learn the connection between weight and the instrument used to measure weights, namely, the balance. Children may be familiar with the balance since they may have seen it in the market or in a shop. It is also easy to make a balance with simple materials.

How to make a balance using simple materials.

Children can make this balance with the help of an elder person. You need the following material

- a small plastic bowl or a plastic lid
- a big rubber band
- a stick
- a needle
- and some thread

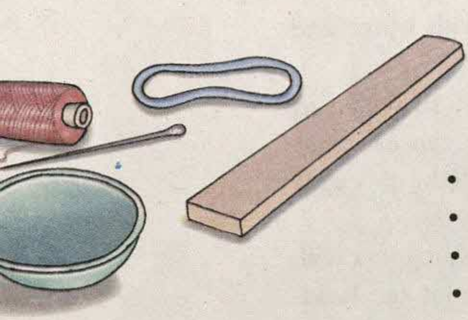


Fig. 4.16

1. Make three holes in the plastic bowl and tie the threads as shown in the figure.

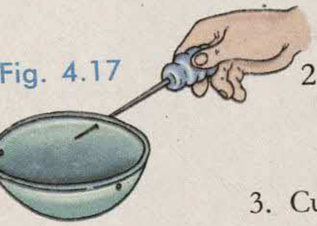


Fig. 4.17

2. Make a knot on top so that the bowl hangs flat when you hold it by the knot. Your weighing pan is ready.

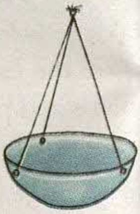


Fig. 4.18

3. Cut open the rubber band and tie one end to the knot.
4. Tie the other end of the rubber band to a stick and hang it from a table or a window ledge.

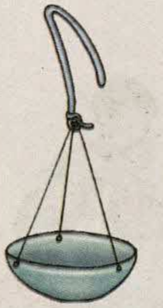


Fig. 4.19

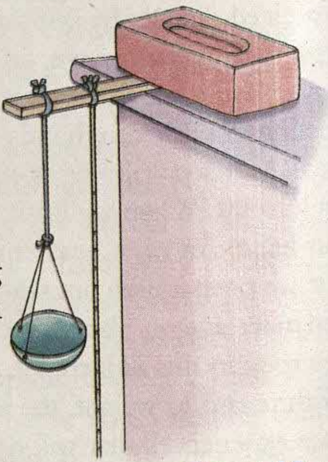


Fig. 4.20

5. Tie a small stone to a string and hang it next to the pan like in the picture.

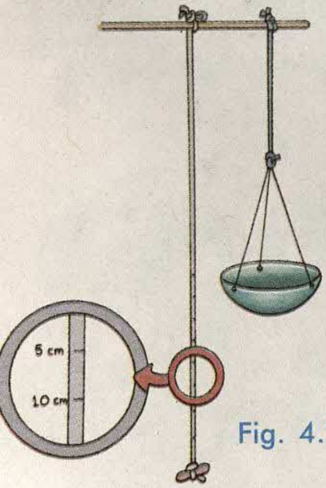


Fig. 4.21

2. Mark the position of the pan on the string with a pen. The string helps you to check how much the pan goes down.

   Put a small stone in the pan. See what happens?

   Then put a heavier stone in the pan and see what happens?

   (Reference: Maths for every child, class 3, Homi Bhabha Centre for Science Education)

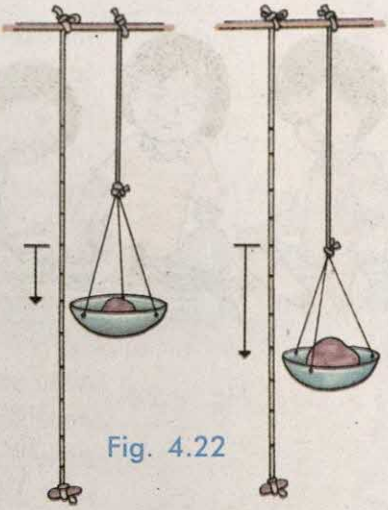


Fig. 4.22

Some of the Activities that can be done are—

1. Take two identical small sized buckets. Fill one of them with water. Fill the second bucket with stones or with sand. Ask the children to lift each bucket in turn. Are they surprised by the weight of the second bucket? Ask them to describe what they felt. Encourage them to use the language of comparing weights: heavier / lighter, has more weight/ less weight.

Fig. 4.23

2. Take three identical plastic balls. Make a hole in each of them. Fill the first fully with water and seal it. Fill the second fully with sand and seal it. Fill the third with sawdust and seal it. Ask the children to lift the different balls. Are they surprised? What did they experience? Can they guess what is in each one of them? The activity again gives them a chance to use the language of comparing weights: heavier, heaviest, lighter, lightest, etc.
3. Bring a shopkeeper's balance to the class. For this activity, you need a strong balance. Prepare different weights by packing sand in small and medium sized plastic bags. If plastic bags are not available, use paper bags or cloth bags.

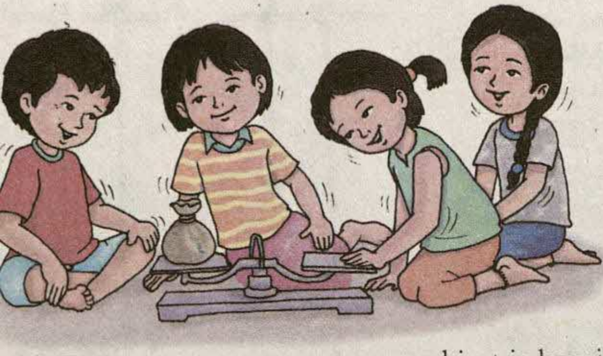


Fig. 4.24

You can tie the bags with a rubber ban Ask the children to put a bigger weig in the first pan and keep the second pa empty. Ask one of the children to pre on the empty (second) pan with he hand gently so that the first pan is lifte As the first pan goes up, add anothe large weight to it. What happens? Wha should they do to lift the first pan if th object is heavier? Give a chance for all the children t try lifting the first pan by pressing on the second pan wit their hand gently.

4. After doing Activity 3, ask them to add some small weigh in the second pan which was empty, but still not enough t lift the first pan. Now tell them to press the second pan agai What do they notice? Let them add more weights to th second pan. As they add more weights does it become easie to lift the first pan by pressing? As they go on adding th weights in the second pan, at some point the first pan is lifte up. Discuss what happens here with the children an introduce them to the appropriate language: 'equal weigh 'balancing the weight'.
5. When the weights in, the two pans are balanced, ask th children what will happen if the weights in the two pans ar interchanged. This will be quite interesting to the childre After they make their guesses, interchange the weights an discuss what happens.
6. For this Activity, you may need a more sensitive balanc Take a lump of modeling clay (or soft doug made from atta) and put it in one pan. Tak another lump and ask one of the childre to adjust the amount so that it balances th first lump. Now take the first lump an break it into two smaller lumps. Ask th children what will happen if you put it bac into the pan. Will the two pans still b balanced? What will happen if the lump broken into many small lumps?

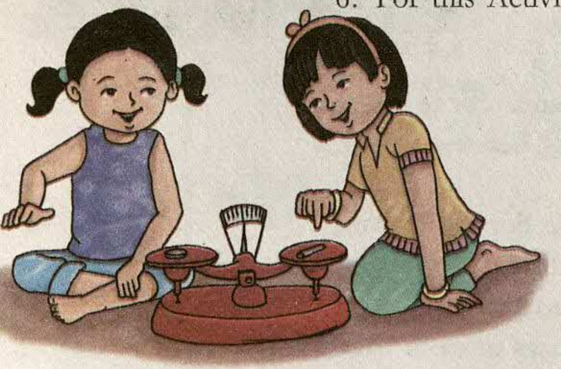


Fig. 4.25

7. For this activity, you can use a sensitive balance. Children can use one rupee or two rupee coins as standard weights. They can guess how many coins will balance small objects from their classroom: duster, chalk, notebook, etc. They can then check this by measuring with the balance. Introduce to them the vocabulary of measuring weight: 'weighs as much as two coins', 'is as heavy as 5 coins', etc. They can then order the small objects that they weighed in terms of their weights.

As in the case of volume, with regard to weight, the main aim is to give children experiences on which they can build later. The focus is on giving them a feel for the attribute of weight and to make them familiar with the language of comparing and measuring weights. The balance is the main instrument used to compare and to measure weight, and they learn how to use the balance by working with informal, easily available units of weight.

ASSESSMENT

Measurement is a topic that is inherently activity based. There is plenty of opportunity for children to actually measure and to work in groups. We can encourage this. Measurement involves both understanding and skill. The teachers can look for the following steps in the children's growing understanding of measurement.

- Understanding and using the language of comparison. Use of appropriate attribute words for different situations.
- Comparing and ordering three or more objects according to a specified length attribute.
- Making comparisons when direct comparison is not possible – by visual estimation, and by using an intermediary object, by using a container etc.
- Making comparisons by using informal units such as handspans, cups, or coins.
- Answering and asking questions of how long/how short? How heavy, how much water etc.
- Expressing the attribute in terms of the informal unit used.

- Recording the measure of different attributes and writing down the number in terms of units.

Skills include finding and using a unit, measuring the attribute accurately, keeping track of counting the number of times the unit is applied, and estimation skills, such as comparing virtually or by feel.

Data Handling

5

IMPORTANCE OF DATA AND INFORMATION

Fig. 5.1

Present age is an age of information, and the average person has much more access to it than before. A lot of information flows through various means of communication like newspapers, magazines, radio, television, internet, etc. As adults, we have to learn to handle information, to interpret it and to critically analyse it. For many important decisions, we need information. A farmer has more control over the price at which he sells his produce if he has information about current market prices. Wage labourers and workers need information about current wages in different cities and industries, and about the cost of living for more effective collective bargaining. A village community can ask for better health facilities and schools if they have information about the public funds allocated to their district and to their village.

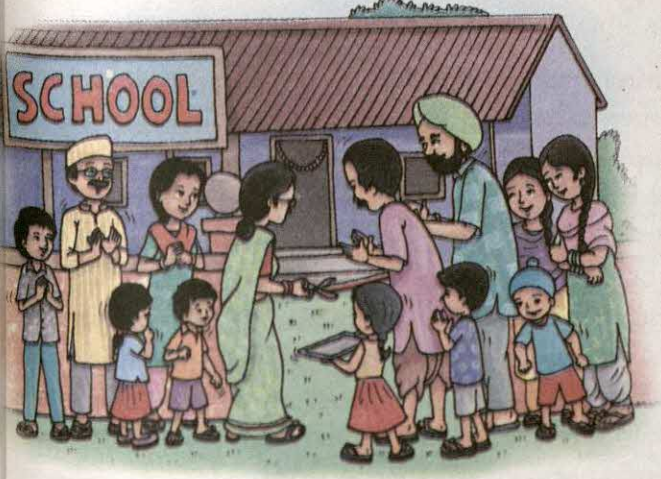


Fig. 5.2

All this information is based on data. 'Data' means almost the same as 'information', and refers to the information in a raw form which is collected from various sources. As the examples above suggest, the capacity to handle and process information and data can be a factor contributing to the quality of living. When we digest information, and turn it into a more general or abstract form, it becomes part of knowledge. Knowledge gives both a sense of well being and power. Information is often manipulated by people with vested interests. For example, advertisements often contain partial information and mislead the customer. In such situations, having access to data and the capacity to interpret data can be a source of power. The capacity to seek, obtain and analyse data does not develop in a short time and is one of the important long term outcomes of education.

The availability of data, which is reliably and systematically collected, makes a system transparent. This is important for a democratic society. It is only when people have confidence in their own capacity to handle and interpret data that they will also seek data. Only when people seek data will society or the government or other authorities set up mechanisms for collecting or providing data and information. A landmark legislation in our country is the Right to Information Act (RTI), which empowers ordinary citizens in having access to data and information that is important for their lives.

Why do we collect data?

We collect information and data when we need to answer a specific question a problem or when we want to understand a situation in generality. This may be because we need to make a decision. For example, the teacher may collect information on the different activities or games liked by the children along with the reasons behind their liking. This information can help the teacher in designing activities for the children. If data on children's attendance is collected over a period of time, one might see patterns of rising and falling attendance on different weekdays. This could help a teacher to make a decision about whether to hold an important activity like a test on a Monday.

It happens often that people have opposing views on important issues. In such situations, it may be critical to have access to the correct information. Thus data not only help us to understand the situation or problem in more depth but also helps to resolve conflicting claims.

Here is an example of data taken from the Eleventh plan document. It is from a working group report prepared by the Department of School Education in the MHRD ministry. The data, collected over three All India Educational Surveys, is presented in the form of a table and pertains to access to schools in rural areas.

Table – Access to schools in rural areas

Particulars	1986 (5th survey)	1993 (6th survey)	2002 (7th survey)
Rural Population	593560310	659691045	742490639
Rural Habitations	981864	1060612	1231391
Habitations having primary schools/sections within 1 km	823117	884089	1070863
In percentage	83.83	83.36	86.96
Population served by primary school/sections up to 1 km	560622974	618543482	731574982
In percentage	94.45	93.76	98.53
Habitation served by Upper primary school/ sections within 3 km.	726594	807656	961899
In percentage	74	76.15	78.11
Rural Population Served by upper primary school/ sections within 3 km.	498447378	560769559	639561298
In percentage	**83.98**	**85.00**	**86.14**

What questions can be answered from the above data? We can, for example, compare how access to primary and upper primary schools has improved over a 15 year period. We can know that even in 2002, about 22% of the habitations did not have an upper primary school within 3 km. By comparing with the population percentage, we see that these 22% of the habitations contained only 14% of the population. So these habitations are smaller in terms of population. This may lead us to further questions. What are the special problems of smaller habitations? Can there be a different arrangement to provide schooling access for children from such habitations such as school buses, or residential schools?

Thus we see that the data answers some questions, while at the same time it raises further questions, which cannot be answered from the data. Other questions that the data may raise are, does the increase in access also mean better quality education for children? Does access ensure that children actually attend school? One can also ask questions about the data itself. For example, how reliable is the data? What were the methods used to collect the data?

There are many examples of such data which you would have come across in newspapers, articles and journals. These are not just numbers; they are meaningful and convey information. The word data literally mean 'given'. Although data can be in different forms—text, spoken words, pictures, etc., commonly it is in the form of numbers, that is, information which is systematically collected and can be quantified or described in numbers.

Data collection and handling is usually thought of as a part of statistical activity and so only of interest to people specialising in statistics. We rarely acknowledge the fact that in everyday situations we are collecting and using data. A teacher is collecting data even when she takes the attendance of children in her class Let us look at another example of data collected by a teacher.

A teacher of Class V gave a mathematics test to 22 students in her class and systematically analysed their response and tabulated the data as shown in the following table.

	Competencies	Children's Responses		
		Correct	Incorrect	No Response
1.	Reading and writing numbers	15	5	2
2.	Finding the Place value	9	4	9
3.	Ordering of numbers	3	8	11
4.	Double digit addition without carry over	22	0	0
5.	Double digit addition with carry over	17	5	0
6.	Three digit addition with carry over	17	5	0
7.	Double digit subtraction without borrowing	21	1	0
8.	Double digit subtraction with borrowing	16	5	1
9.	Subtraction from zero	12	10	0
10.	Operation sense for word problems	1	6	15

The teacher was able to find out which competencies were not yet attained by the children. From the table the teacher realised that although many children could read 3-digit numbers and could tell the place value, they were not able to compare the numbers.

This suggested to her that they had learnt place-value in a 'rote fashion' without relating it to magnitude or size. She also noticed that some of her students who could do the subtraction with regrouping had a problem when there was a 'zero' digit in the top number (subtrahend) she decided to pay more attention to subtraction with regrouping. But the biggest problem was with word problems – only one student could identify the correct operation for a word problem. She wondered if she should change her teaching approach in some way.

What suggestions would you have for the teacher?

Children too are often seen generalising and interpreting information in everyday situations. For example, they may have opinions about who eats the most *chapatis* in their house, which game is most liked or disliked by children and so on. But such opinions are usually based on isolated observations or random impressions. If a child has seen her younger brother eating more *chapatis* one day she might generalise that her brother always eats more *chapatis*. In the school and the classroom, we provide opportunities to children to answer questions that they find interesting, and to check or verify their answers, by collecting data systematically, by presenting the data and interpreting it.

As adults, we would be interested in particular kinds of data but for children such data may not be interesting. We need to think of data which is meaningful and interesting for them, for example, who can jump the longest, who is the tallest, etc. For children in Classes I and II, the data should be from their immediate environment. Children like to collect information which is related to them or their immediate surrounding such as their likes and dislikes, familiar objects, family members, etc.

Collecting and using data involves the following steps:

(a) Identifying and framing question(s)
(b) Seeking, collecting and recording the data
(c) Organising and displaying the data
(d) Interpreting the data or making sense of the data

(a) Identifying and framing question(s)

Before collecting data it is very important to identify the question which you would like to answer and to make it as clear and precise as possible. You can discuss with the children and come up with a question that is interesting to them. As a teacher, you can help in making the children's questions more precise. For example, th children may be interested in knowing who is the biggest child in th class. But what does this precisely mean? Is it the tallest child? Or th heaviest? Or the child with the biggest hand? Or the biggest head? By making the question more precise, we make it possible to answe the question and also make the children's thinking conceptually riche

When we ask young children about what they like, fo example, their favourite food item, then we may not get consisten responses. When a teacher of Class I asked the children wha their favourite food was, she found that the children kept changin their choices depending on what they were reminded about an what the other children said. She found that the children wer answering more on the basis of their feelings at that moment. I was hard for them to keep their feelings aside.

(b) Collecting and recording data

After identifying the question that the children want to find an answer to, the next step would be to collect the relevant information or data in order to answer it. Various means can be used to collect the data such as observation, interviews, and questionnaire. Young children can collect data in the following ways.

(i) **Observations** – Children of Classes I and II would be collecting data mostly be observing. Such data may be quantitative and hence involve counting and measurement. For example, observing the number of vehicles passing by the school gate, observing different flowers or shapes of the leaves in the school campus, finding out the tallest child of the class, finding out who has the biggest feet or nose, how many times they can throw the ball up and catch it without letting it fall down, etc.

(ii) **Finding information by asking questions** – Children can also collect information by asking questions to their friends, family and others. They may find out the number of pets or animals at home, the number of family members, etc. Obtaining information by asking questions may help in strengthening their communication and interpersonal skills. Children can also ask their friends what they had for breakfast, what they like and dislike, etc. Here the teacher needs to be sensitive as mentioned above since young children may not give consistent responses to questions about their likes or dislikes.

The data collected should be recorded in a systematic way so that it can be read easily and clearly by others. There could be many ways of recording and presenting the data. With Classes I and II children, the recording process should not only be easy to carry out but also meaningful and interesting. To record data, children can use concrete materials like stones, seeds or bottle caps, making marks such as smilies, tally marks or tick marks, drawing icons and objects, etc. Let us look at some examples of activities and see how these different ways of collecting and recording data can be used.

Why do we need to record data?

- The data might be too large and if we do not record it we might lose some data.

- We cannot rely on memory alone, since our capacity to remember is both limited and not very reliable.
- We might like to use the data later. We might like to compar data collected at different times.
- We might like to share it with others.
- We might like to put together data collected by different peopl to see the larger picture.

Different ways in which children can record data

- pasting / putting concrete objects on chart paper / floor.
- representing data through drawing icons.
- using strips
- dots/ sticks/stars can be used as tally marks instead o numbers.

Example 1: Vehicles passing by the school gate during lunc break

The children decided to find out the number of different vehicl that pass by the school gate during lunch break. This activity w carried out by children of Class I. Each Class I student was giv a specific task. One child had to watch and count the bicycl that passed by the gate. Another child had to count cars. A thi child also counted bicycles, so that she could check her data wi the first child's data. A time period of 15 minutes was given f counting the vehicles on the road. It was very interesting to s that each child was doing the task assigned to him or her ve seriously and diligently. Also they found their own ways of keepi a record of their counting. For example, one of the children w using his fingers for keeping a record. Another child was makin marks on the copy she was carrying and so on. After the studen came back to the classroom, each one was asked to talk about h or her experience. This was quite interesting. One student shar that he got confused when two cyclists were going together should he count them as one or as two? The class listened to ea other's experiences and discussed it. The teacher drew simple li figures of the different vehicles on the board and asked the childr to write the numbers below the pictures. When a child raised question about which vehicle was seen most frequently, t students started comparing the numbers. One of the students ma a guess that bicycles were the most frequent. This was verified by the other students with the help of the data. Other questions were raised and comparisons were made by the children. Thus, the children were making sense of the data that was displayed in front of them on the blackboard.

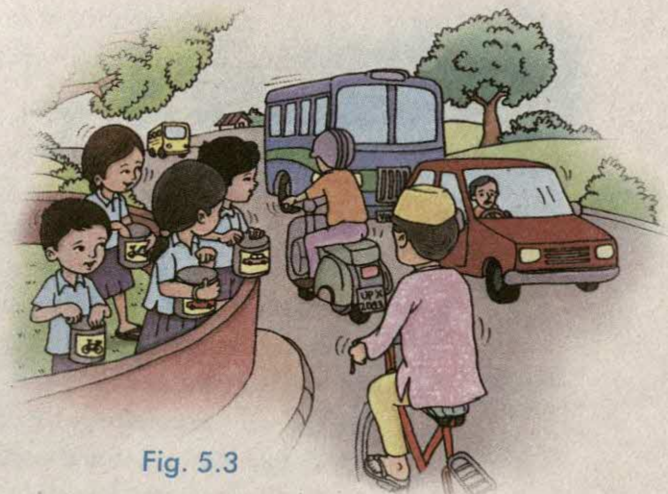


Fig. 5.3

In this example, the children could have kept track of the number of vehicles in a different way. Each child could have been given a container with the name or picture of the vehicle pasted on it. For each passing vehicle, a stone or a seed could be put into the container. Finally, by counting the stones in the container, the students would be able to find out how many vehicles pass by the gate during the 15 minute period.

Can you suggest some other way of collecting and recording data for this activity?

____

____

Example 2: Jump as far as you can

This is a game in which the children try to jump as far as they can and land on the various shapes shown in Figure 5.4. Each child records his or her jump by placing a stone on the boxes next to the shape on which they have landed as shown in Figure 5.4.

At the end, children can count the stones on each shape and can discuss questions such as

- How many children have reached the farthest shape?

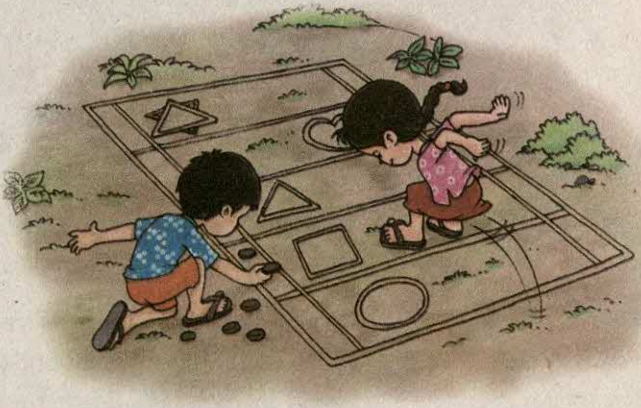


Fig. 5.4

- How many children have only reached the first shape?
- Which is the shape on which most children have landed?

***Example 3*: Favourite sweet**

We have discussed above how it is difficult for children to choose and name their favourite food. One teacher got over this difficulty in an innovative way. She got different bowls and pasted the name of a sweet on each bowl. Each child was given a pebble. The children 'voted' for their favourite sweet by putting the pebble in the correct bowl. The votes cast for each sweet was counted and recorded on the board and the favourite sweet was declared at the end of the counting. Based on this 'voting' data, the teacher got different sweets for the children on the following day.

In the last two examples, we see that in the process of collecting data, information about the jumps or choices of individual children is lost. In the jumping game, since all the children used stones as a record, we do not know which child jumped on which shape. This is also the case in the example of 'Favourite sweet'. However, we can get this information by asking each child to use, instead of a stone, a tag with his or her name written on it. The teacher can keep such tags made from thick cardboard with each child's name written on it. They can be used for different activities involving data collection.

Children should also be encouraged to share the process by which they collected the data and the challenges they faced while collecting it. The teacher can initiate such a discussion. A discussed earlier, children had difficulty in recording when tw vehicles appeared at the same time or were moving fast. Suc discussions help the children realise the importance and difficult of recording the data reliably and to develop the necessary skill Children can also be encouraged to discuss how to collect an record the data, *prior* to the data collection. The following exampl illustrates how this might happen.

***Example 4*: How big is your head?**

A Class II child claimed one day in the class that he had the bigge head in the class. This triggered the activity of measuring ho big everybody's head was. The children discussed various way of measuring the size of the head. Finally, with the help of the teacher it was decided that the children would form pairs and measure the circumference of their partner's head with a paper strip. They would take a long enough paper strip, take it round their partner's head and tear off the excess length of the paper strip. Then the child's name would be written on the paper strip so that she could identify it. After the children had finished measuring there were 40 to 45 paper strips and the question arose of how to compare the measurements. Again many suggestions were made by children and some children came up with the idea that one end of the paper strips have to be in one straight line so that the other ends could be looked at and compared. So the children were able, on their own, to come up with an idea of measuring the size of the head and of displaying this information so that the sizes could be compared. The display looked like a long bar chart. Each bar represented the circumference of a child's head and it was easy to look at the big bar chart and find out whose head was bigger.

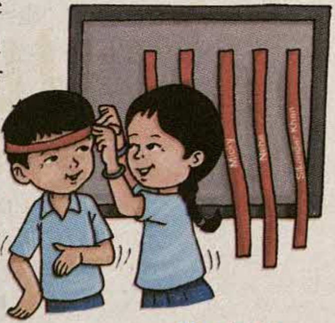


Fig. 5.5

(c) Organising and displaying the data

Once children have experienced, observed or collected information then there is a need to put the information together and to present it so that is meaningful and easily comprehensible. We saw how this was done in the previous example of measuring the size of the head. In the example of the vehicles passing the school, the teacher will need to help the children to pool their data, check if the data between groups counting the same vehicles tallied, and

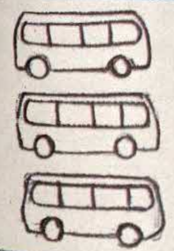
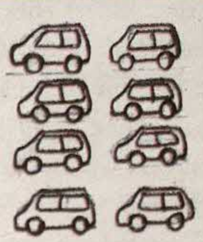
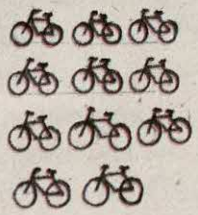
No. of buses	No. of cars	No. of cycles
		

Fig. 5.6

then to decide how they would display it. They could use the blackboard or chart paper to display the data, for example, by drawing the vehicle and writing the number next to it. If the numbers are manageable, they might also decide to stick one picture for each passing vehicle on the chart, ending up with a display called a 'pictograph', which is shown in Figure 5.6.

(d) Interpreting or making sense of the data

Whenever the data or information has been displayed, a discussion can be held in the class about what one can know from the data. The teacher can make a link to the questions asked before the data was collected and see whether the data helps answer the questions. From the displays presented above, the children could easily find out which vehicles are more frequent. This might lead them to ask further questions such as, why there are more vehicles of one kind than another, or if there will be more vehicles seen in the evening or morning. Sometimes the activity of data collection can lead to discovering and sharing related information as the example below indicates.

Example 5: Whose feet are the biggest?

When there was a discussion about whose feet are bigger in Class II, the children realised that they are wearing shoes o different sizes. They also discovered that every shoe has a numbe which denotes its size and that this number can be seen on th sole of the shoe. The teacher explained to the children that th shoe numbers for children's shoes are different from the sho numbers for adults' shoes. The children's shoe numbers are fro 1 to 13 and after that the adults' shoe numbers start. Since th shoe numbers gives an idea of the size of the feet, the class decide on the activity of collecting data about shoe sizes. It was a ver noisy scene as children started looking at each other's sho numbers noting them down. Within about 20 minutes, they ha collected information about the shoe sizes of all the 45 childre and knew how many children had which shoe number. Then th teacher asked some of the children to write it on the blackboa so that all the children could see the data collected. It was foun that most of the children were wearing shoes of size 11 or 1

But 3 or 4 children did have adult shoe sizes. Some children had size 9 or 10. The children themselves started interpreting this data and came out with various conclusions. Some children said that although they were wearing a particular shoe size, their feet were smaller than that size. One child showed the space between his toe and the shoe by pressing it. Their parents had bought a size bigger than their feet for them. This was a common practice because children grow so quickly that they quickly grow out of their shoes! Putting together the data on shoe sizes and organising it made it possible to interpret the data. The discussion that followed gave children the opportunity to relate their experiences with the data.

It needs to be borne in mind that for young children, there is no need to separate out all the steps of collecting and using data. The activity can be made more efficient and easier for the children to do if some steps are combined. For example while recording the data, the children can also organize the data in a meaningful form. To take the example of the passing vehicles again, children can be given cutouts of bicycles or cars. They can drop one cutout of a car or a bicycle in a bowl every time they see a car or bicycle respectively. Finally the cutouts could be stuck on a chart for display.

ASSESSMENT

The points of assessment in relation to data handling are skill in collecting data, motivation and enthusiasm for the activity, participation in discussion, raising questions, making interpretations, etc. It may be difficult to carry out a complete individual assessment for all children. It is better if the teacher notes down in an assessment book, or in the student's notebook, as and when each child makes a significant contribution. It is possible to do group assessment on this topic when students work as a group and collect and present data.

ADDITIONAL ACTIVITIES FOR CHILDREN

1. We have discussed the activity where children measured the sizes of their heads. Children find activities related to measuring parts of the their body interesting. So similar

activities on measuring height, length of arm or leg, size of hand or feet could be taken up. For measuring their feet or hand a graph paper or square lined paper can be used. The children can trace their feet or hand on the graph paper. They can then find the length by counting the squares along the maximum length.

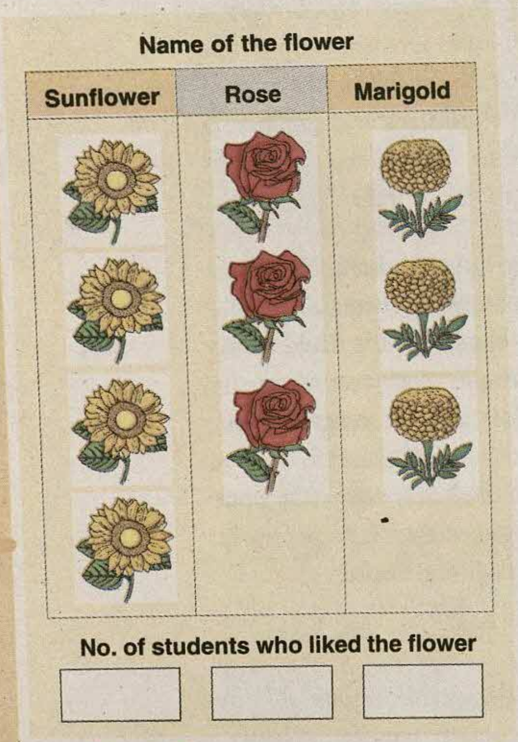


Fig. 5.7

2. Collecting flowers: The children can be asked to bring a flower that they like the most to class. (If flowers are not available, a suitable substitute could be used.) The teacher can initiate this activity by asking the class to guess which flower is liked by most of the children. Let the children guess before they actually find the answer as it makes them more interested in the activity. The teacher can help the children in arranging the flowers they brought so that they can easily see and interpret the data. One way of doing this is shown in Figure 5.7.

After the children arrange the flowers and see for themselves which flower is liked the most, the teacher can take the discussion further. The students could discuss the colours of different flowers, the number of petals, their smells, etc.

3. The children could collect data about animals they have in their house and the data could be displayed in a manner similar t the flower activity. This activity can work well in rural schools where homes have a larger number and variety of animals.
4. The children can collect data about what they had fo breakfast, what transport they used to come to the schoo (auto, bullock cart, bus, train, rickshaw, etc.).
5. Children can find out about the story books they have rea from the school library. The teacher can discuss with th children and select the five most popular books. The childre

could find out how many have read each book and see which book is liked the most.

6. Children can collect data on their hairstyles – how many of them have pony tails, side partition, middle partition, etc.

SOME ACTIVITIES FOR TEACHERS

Teachers may find it interesting to do these activities in a teacher workshop. The activities orient them to the importance of collecting and using data, and of the care that needs to be taken while doing so.

Example 1: Number of children in the class

Often questions arise in teacher workshops about some aspect of schooling, teaching, etc. Sometimes one teacher may take one view, while another may believe the opposite. Such a situation is a good opportunity to collect and use data. Indeed, one of the reasons why we collect data is to decide between opposing claims. The teachers become involved in the activity if it relates to their own views and opinions.

In a teacher workshop, one of the teachers said that there were more children in his class because the village where he was teaching was far away from the town. This led to a discussion about whether it was generally true that schools which were far away from the town had more children in the classrooms. The teachers decided to examine this carefully. So they collected data about the number of children in each teacher's class and the distance of the school to the nearest town. They made a table for each school giving the number of children in the class and the distance in columns. By studying the table carefully they concluded that it was not generally true that the classrooms had more children if the school was farther from the town.

Example 2: School attendance

Every school has an attendance display board which shows the number of children present and the number absent in each class on a particular day. Can you suggest a better way of displaying the attendance so that anybody who looks at it gets the information clearly and easily? The teachers can think of the attendance boards in their own schools and discuss how it could be improved.

Patterns

6

In the child's life, the sequence of daily activities forms a pattern that the child can recognise. For example, the child wakes up in the morning, brushes his or her teeth, takes breakfast and goes to school. In the school, after morning assembly there may be five periods before the lunch break and four periods after the lunch break. This is a regular feature and so it forms a pattern.

Fig. 6.1

In clothes—sarees, shirts, bedsheets, etc. and in many objects around us we can find patterns or decorative designs. They may be in the form of boxes, checks, lines, circles, etc. Children often identify similarities in the patterns and say, 'this design/pattern is on my dress' or 'my father has this type of shirt' and so on.

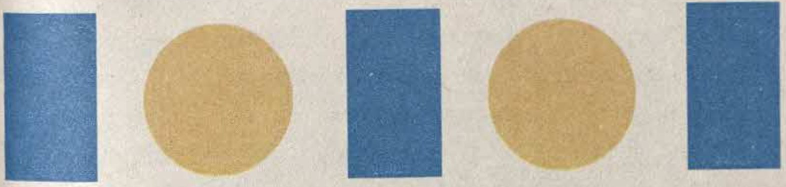
Patterns can be also found in numbers. For example, even the counting numbers have a pattern. Each number is one more than the preceding number. To carry the number sequence forward the child will have to understand the pattern that to get the next number, you have to add one to the previous number. Only if the child knows this pattern, will she be able to predict, for example, that after forty, we get forty-one, and after hundred we get hundred and one.

The word 'pattern' can have different meanings such as decorative designs, figures, motifs, shapes, etc. It can also mean a regular arrangement, order, sequence or repetition. The second meaning is more important in mathematics. Arrangement, repetition and order are important in many branches of mathematics and hence, patterns cut across mathematics. Some people call mathematics the science of patterns and relationships: *'mathematics is the classification and study of all possible patterns'* (Warwick Sawyer, 1955). So it makes sense to include opportunities for students to explore patterns, which also encourages creativity and thinking.

Let us try to understand with some what the help of patterns are examples:

(a) 1, 3, 5, ____, ____, ____

To find out what the next number is, one needs to recognise the pattern. The pattern could be, go to the next number by skipping one: after 5 skip 6 and go to 7. The pattern could also be described as, add two to get the next number: add 2 to 5, to get the next number 7.



(b) In this sequence, children will need to recognise that the circle and the rectangle are alternating, to predict the next shape.

Human beings, even children, depend a lot on patterns in making sense of and understanding the world around them. We make sense of new situations by seeking or even imposing patterns and structures upon them. For example, Ramesh observes his father when he returns from work. He knows the pattern followed by his father: removing slippers, changing clothes, giving him snacks, drinking tea and getting dinner ready. If one day father comes home and does not change his clothes, the child asks questions like—"Papa, are we going out somewhere?" or "Is some guest coming home?"

Fig. 6.2

What does the preschool child already know about patterns?

- Preschool children often arrange toys or cars or dolls in a straight line and later perhaps in two or three parallel lines.

Fig. 6.3

- If similar objects are placed alongside each other in a repetitive pattern, this might indicate appreciation of symmetry.
- The child may arrange objects in particular ways for specific purposes, for example, to count, to highlight properties like colour, to compare length, etc.

Why should patterns be taught?

Patterns have not been taught as a topic in school mathematics traditionally. So why should they be taught? Some reasons are

- Patterns are attractive and are found all around us. So they are engaging to children and adults. Describing a pattern increases one's awareness and understanding of patterns. Working with patterns encourages the child to observe and identify designs, shapes, numbers, etc. around her.
- When asked to look for patterns, the child may observe similarities, dissimilarities, repetition, non-repetition, etc. and discover a pattern, which otherwise she may not have discovered.
- When a child is asked to predict what the next element in a sequence is, she becomes more attentive to and observant about patterns. Also, predicting, checking, verifying are important aspects of mathematical thinking.
- The child may learn to superimpose one pattern on another one or compare one pattern with the other. This allows the child to move to a more abstract level, which is again an important part of mathematics.
- In general, working with patterns helps to develop the child's reasoning.
- In future, it will also help in understanding algebra.

There is a need to expose children to a variety of activities involving patterns. Of course, we have to keep in mind that the activities should follow the principle of moving from simple to complex, known to unknown, concrete to abstract, etc., so that children get a feel of the activities, start enjoying and get involved in discovering patterns in the environment around them.

Suggestive work on patterns for Classes I and II

1. Recognising and Describing Patterns

The first step naturally is observing and describing patterns. Here one can start with simple patterns, such as those based on one simple characteristic and move to slightly more complex patterns. Many patterns are found around us like the ones in the picture below.

Different children may look at the same pattern differently and verbalise or explain the pattern in different ways.

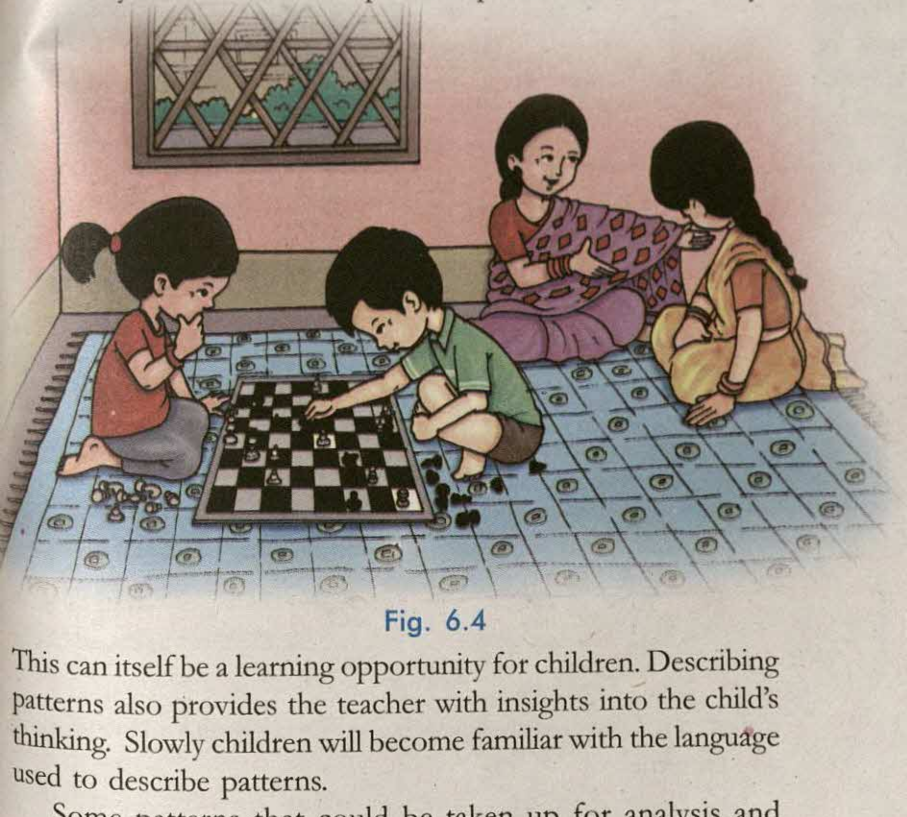


Fig. 6.4

This can itself be a learning opportunity for children. Describing patterns also provides the teacher with insights into the child's thinking. Slowly children will become familiar with the language used to describe patterns.

Some patterns that could be taken up for analysis and discussion are as following:

(i) Designs on fabrics
(ii) Designs on tiles of floor

(iii) Prints on shirts

(iv) Pattern on the window grill or curtain

(v) Chessboard, etc.

2. **Copying and Continuing Patterns**

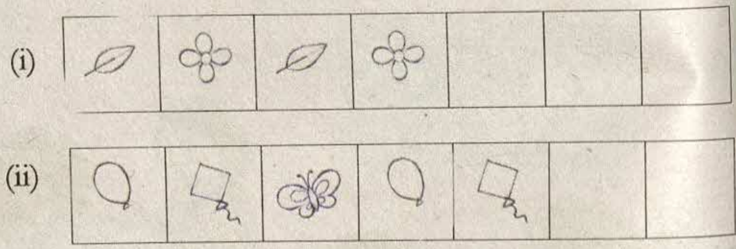
Repeating patterns, contain a unit which repeats. For example, in the chessboard, the repeating unit is a rectangle containing one white and one black square. Once the child has recognized the 'unit of repetition' in a pattern, she can be encouraged to copy or continue the pattern. Some simple examples of repeating patterns are shown below.

A. Identify and draw the 'unit of repetition' in each pattern.

B. Copy and continue the pattern

Note for Teachers

Do not use the word 'unit of repetition' with children or make them identify the 'unit of repitition' explicitly. When they continue a pattern they implicitly understand the 'unit of repitition' and this is sufficient.



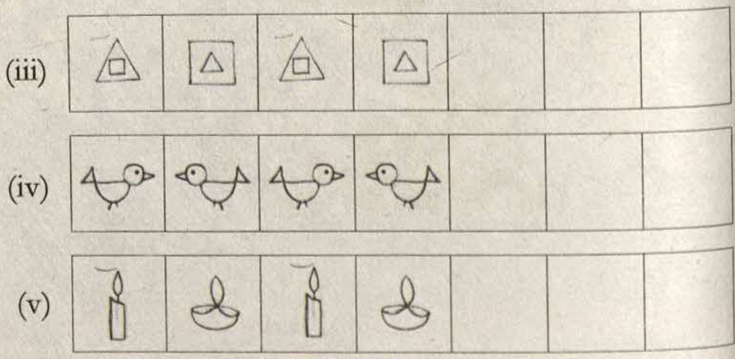


Fig. 6.5

These activities provide opportunities for peer learning, for children to learn from each other while copying and continuin the pattern.

3. **Creating Patterns and Free play**

Children love to explore, analyse, extend and create pattern while playing with objects. *Bindis*, stones, leaves, threa matchsticks, buttons, flowers and bangles can all be used by children for making patterns as a part of their play activity. We can use the skill of young children and incorporate it in our teaching-learning activity. We have all observed that children given space and material come out with many beautiful and creative patterns, designs sequences. Some of them are shown in Fig. 6.6.

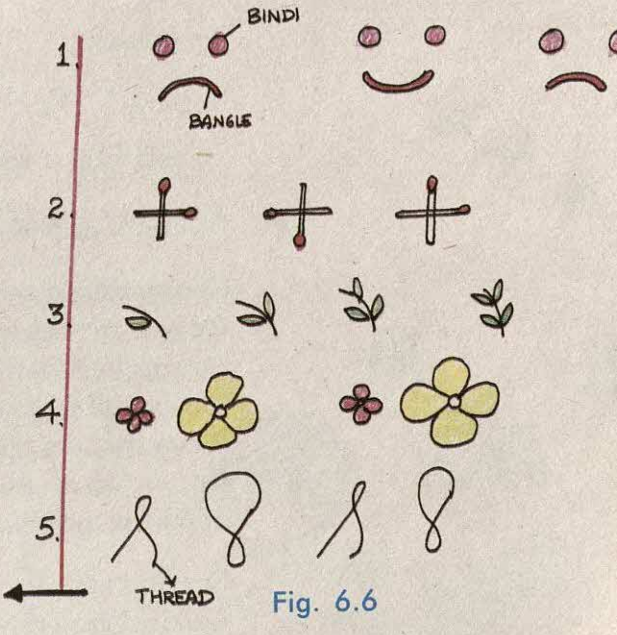


Fig. 6.6

The learning kit has stamping containers which can be used by children to make different patterns of their choice. Fig. 6.7 shows the work done by Class II child.

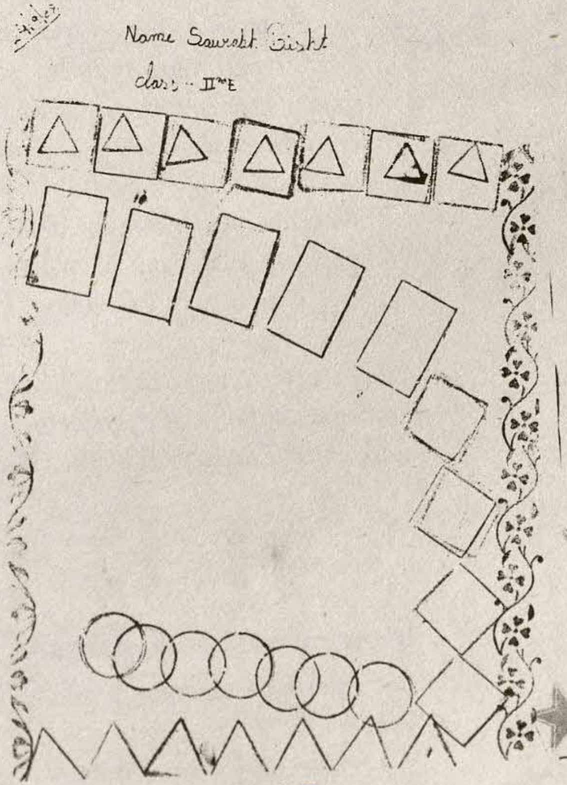


Fig. 6.7

For children in both Classes I and II, we can give plenty of opportunities to make patterns using objects around them. Gradually, we can move on to making paper-pencil patterns. Children are fond of making patterns by cutting coloured paper. If coloured paper is not available, we can use newspapers which are easily available. Children also love making patterns with their finger-prints. Patterns need not be restricted to visual patterns. Children enjoy clapping and they can create beautiful patterns out of clapping.

For example,

clap – clap – clap –

clap. clap – clap. clap – clap. clap –

clap – clap. clap – clap – clap. clap –

4. **Comparing patterns**
   We can ask children if there is any similarity between the patterns in A(i), B(iii), B(iv) and B(v) on the previous page. They are all examples of the alternating pattern—*a b a b a b...* When children notice the similarities or differences between patterns, they are working with abstractions, which is an important preparation for mathematics.

5. **Sharing and discussing patterns in a group**
   Children can create these patterns in groups. We know that group work helps us learn from each other—we can share our ideas, can see the same pattern from different perspectives and can learn to accept the perspective of a friend. While helping a friend complete the pattern, children often clarify their friend's or their own understanding. Group discussions could be followed by presentation of the patterns that have been found in front of the whole class, this would help in making the learning more effective.

Children can also make pattern sequences for other children to complete. Another interesting problem is to create a **non-exemplar**, that is, an element that does not fit in the pattern, and ask children to find the odd one out. For example,

↑↑ ↓↓ ↑↑ ↑↓

More examples of patterns

Patterns can also be made with numbers. Here are some examples.

(i) Pattern for one more—
1, 2, 3, 4, …

(ii) Pattern for two more –

1, 3, 5, 7, … or 2, 4, 6, 8, …

(iii) Increasing the gap between each number and the next number –

1, 2, 4, 7, 11, …

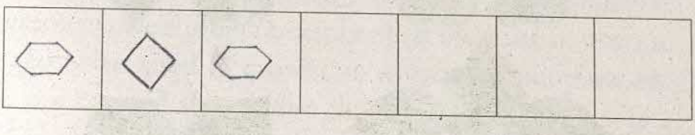
(In this sequence the difference between two numbers is one more than the previous difference)

Children can take these examples further by making their own patterns. They can use square pieces from the learning kit to make number as well as colour patterns. We should only use numbers familiar to children while making the patterns. For example, in Class II children may be able to easily handle numbers up to 50. In some classes teachers may decide to work with only smaller numbers, in other classes the teacher may decide to work with bigger numbers it depends on what the children are comfortable with.

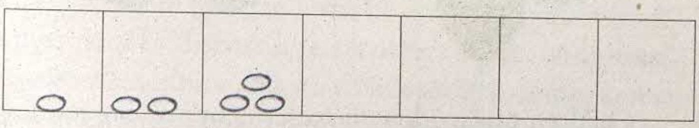
Fig. 6.8

Here are some examples of patterns created using various shapes. For example,

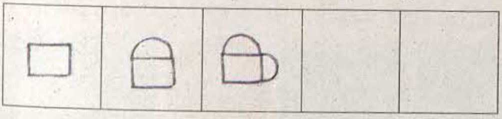
(i) Varying the corresponding shape



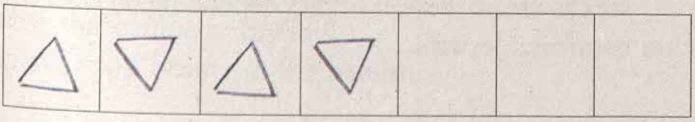
(ii) Varying its number



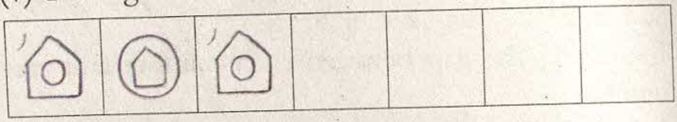
(iii) Adding one attribute to the shape



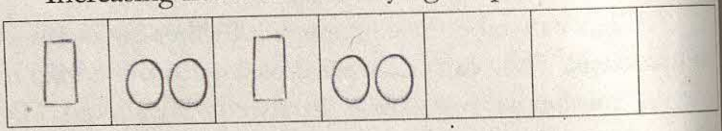
(iv) Moving the shape up-side down



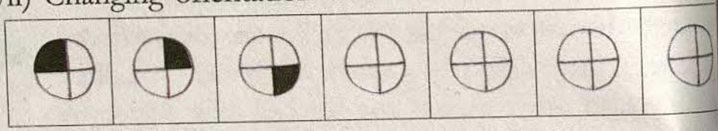
(v) Putting one within another



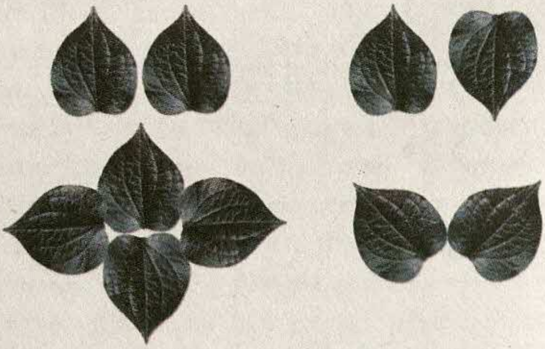
(vi) Handling two characteristics at the same time: Increasing number and varying shape



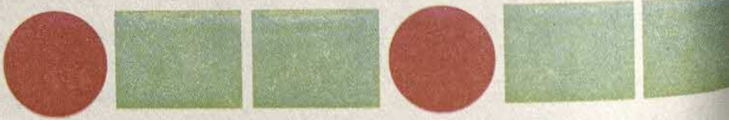
(vii) Changing orientation



In Class II, we can increase the complexity. For exampl we can rotate the shapes to form beautiful patterns:



Children can work with patterns by varying number, shap orientation, etc., arranging them to obtain more attracti patterns.



We can create these examples depending on what the childr are comfortable with.

Patterns in the multiplication tables

While doing multiplication table of 2 with the children, the teacher discovered that one child had found a pattern in the table of 2. When asked to explain, the child explained that the unit's place has only 2, 4, 6, 8, 0, which keep repeating in all the multiples.

	10	20	30
2	12	22	.
4	14	24	.
6	16	26	.
8	18	28	.

Every multiplication table has a pattern, encourage children to discover such patterns.

Points of Assessment

The goal of assessment is to give feedback to the child and to the teacher. Encouraging the child or pointing out something in her work will help the child learn – this is the basic purpose of the assessment. We should remember that there are no formal milestones to be reached or crossed in learning about patterns. We should not be rigid about what children learn. In fact, it is a good idea to use self assessment or peer assessment for this topic, since children are likely to learn by comparing their work with others' work or by discussing each others' work. In a group, children discuss what is nice about a child's pattern or may come up with other interesting ideas. The teacher can invite the children to discuss the work of the members in the group. By asking questions gently, the teacher can make the children focus on the following points.

(i) Similarities or differences observed in the pattern.

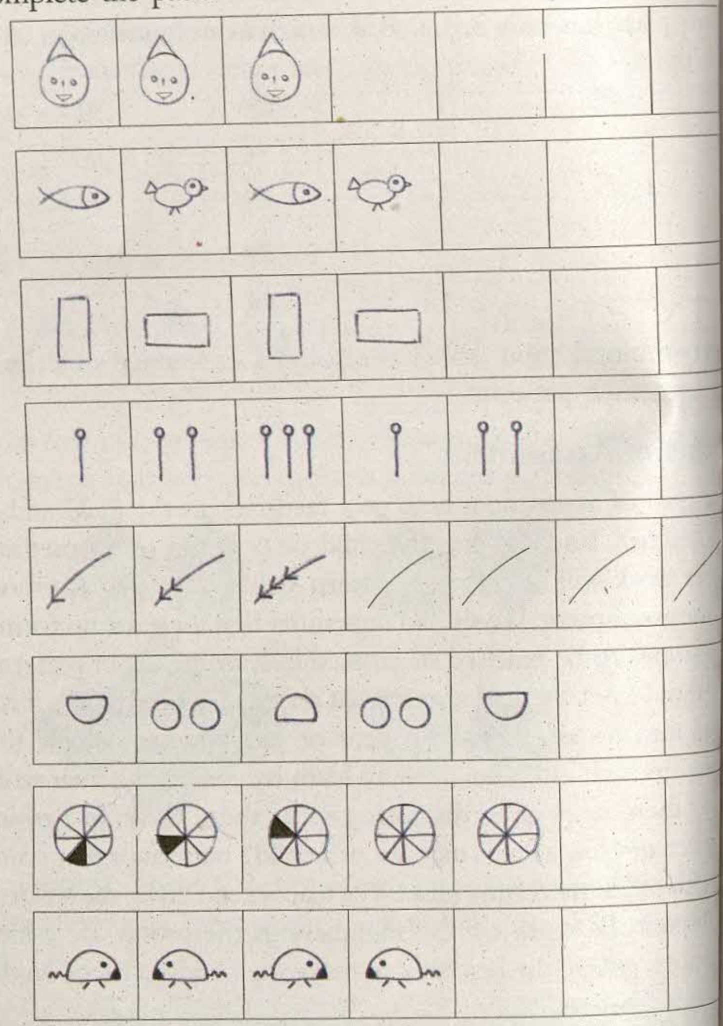
(ii) Finding the key characteristic to move the pattern further. i.e. looking for 'repeating unit'.

(iii) Drawing the complete pattern.

(iv) Complexity and beauty in the pattern.

Sample Worksheet

This is a sample worksheet which includes all kinds of patterns
It can be used as one of the tool for assessment.
Complete the pattern.



Project Work

Children can also make interesting projects focusing on patte
They can prepare scrapbooks with the help of cuttings fr
fabrics, gift papers, leaves, decorative bindis, or other sim
material. They can also look for and find patterns in th
environment, which they can draw in their scrapbooks.

HOW TO ASSESS CHILDREN'S PROGRESS

7

Learning is an ongoing process. Children learn from nearly everything that they see and act upon. It is through the daily experience and interaction with the environment and with other people that the child learns or modifies what she has learnt and makes progress. But the progress made by a child is not always visible and may need to be looked for or probed. At times when we think that a child has not learnt anything, in reality she might have experienced and learnt much more than what we expected or intended.

No child starts walking in a day or two. They take their time to gather the strength, coordinate their motor skills and then to take their first step. A little progress is made each day and this is usually noticed by observant parents. Similarly, Anju a student of Class II, initially used to pick beads every time she had to add or subtract but now after a few months she gives the answers without using any concrete aids like fingers. She has learnt the number facts. This would not have happened in a day but only as she gained experience and practice with addition. To the teacher this shift made by the child may not be visible, but she can look for such developments. What made the child shift? When did she change? This is how a person progresses. **Progress is a step by step process.**

burr... ... ...
burr... ... ...
burr... ... ...

Fig. 7.1

Children's understanding of various concepts develops step by step. It is not that suddenly one day a child will say the number sequence up to 10. It is only by constantly watching and listening to others counting, trying repeatedly and getting corrected, that she grasps the number sequence. To know 'how children learn' is essential for teachers, since it tells us what input is exactly required. This knowledge comes from careful observation of the learner while assessing him or her.

Every learner progresses at her own pace. No two children are the same. They think differently, react to situations differently, come from different backgrounds, value different things, etc. **Each child is unique**. No child who comes to the school is a blank slate. The child brings with her some knowledge on which she builds new knowledge (understanding and experiences.) So the first step towards assessing and understanding progress i to observe the child at the individual level. To assess a child the teacher needs t understand the chil first. Unless you kno what the child alread knows, you cannot sa what new or differen knowledge or skill sh has gained and ho much she has progresse on the path of learning

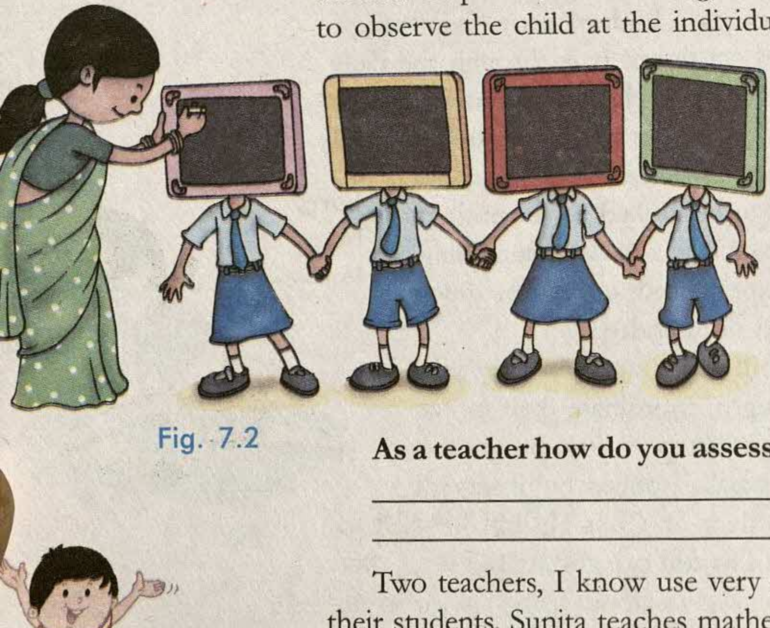


Fig. 7.2

As a teacher how do you assess your children's progress

____

____

Two teachers, I know use very different ways of assessin their students. Sunita teaches mathematics to Class II childre She tries her best to plan her lessons according to h understanding of what her children might already know. Sh builds on the children's existing knowledge. At the term en i.e., after three months, she takes an examination to chec whether she has been able to achieve her teaching learnin objectives and to record the child's progress during the tern

The other teacher I know, Kavita, also teaches Class II children. She does not feel comfortable with the traditional report card, which only provides an impersonal report and cannot describe the complex ways in which children learn continuously. She feels that one should not forget the entire session work of the child and just mark her on the basis of the exam performance. She thinks that it is more reliable if we assess the child's learning in different ways.

She divides each unit or the mathematical concept into sub-units and observes how the child responds to the activities designed for a particular unit. Her assessment is an integral part of her day to day teaching process and not a report that appears after 3 months. This way of assessment helps her in knowing her children's progress, their efforts, their strengths and weaknesses. It also helps her in planning what to do next to help an individual child progress to the next level. To enrich her assessment while writing the report for each student at the end of the term, the teacher adopts a strategy. She helps the child to maintain his or her own file of sample work—**a portfolio.**

Fig. 7.3

The portfolio contains samples from the work done by the child – something that the child wrote or a problem or worksheet that she solved, or a project that was carried out, or even the teacher's observation or appreciation. While writing the report about each child she looks at his or her portfolio.

She feels that this continuous assessment of student's work not only facilitates their learning of mathematics but also enhances their confidence in what they understand and communicate. It also gives her a better picture of the progress of her students.

As teachers, what do you feel about these two different strategies?

In our educational setup we mostly assess children for giving them grades or promoting them to next class. Assessment also has other purposes. One of them is to understand the path that a child is taking and to know where she needs help. However, for this to be possible, the teacher should have a thorough knowledge of how children learn a particular concept. For example, Champa writes mirror images of most of the numbers. However, she can give you seven marbles when she is asked to do so.

Fig. 7.4

Is correcting Champa's mirror writing a cruical step for her to make progress in understanding numbers?

OBSERVING AND INTERACTING WITH CHILDREN

Observing children tells us a lot about how children learn an think. It also tells a teacher what she is expected to do nex Like in a group of five a teacher gave 8 bottle caps to eac child and told them to count and write how many bottle cap each had.

Rubaiya: I have got 7 bottle caps.

Teacher: Let us count again

Rubaiya started counting and while counting she le a bottle cap and so she got 7 as the answer.

(Now that the teacher has identified the proble she knows what is to be done next.)

Teacher: (to Rubaiya) touch each bottle cap you count.

Rubaiya touches each bottle cap but still miss out one since they are randomly arranged. So, teach tells Rubaiya to put them in a line and count. She g the right count.

Another child, Mani writes the numeral 6 on notebook. On being asked by the teacher, he sa

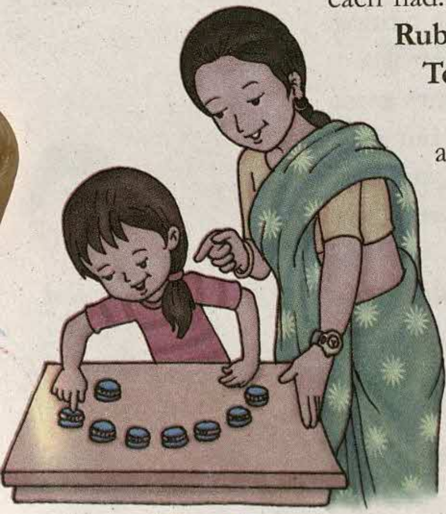


Fig. 7.5

there are 8 bottle caps. When the teacher asked what he has written he said 8 (but he had written 6). It is by interacting with the child that the teacher came to know that he cannot recognise the numerals well. Since the teacher has analysed the error she can identify the appropriate task for that child to do next.

In both the cases if we had assessed the progress only in terms of end results i.e. saying or writing how many bottle caps you have got, then both the students failed to count correctly. However, this is not the complete picture. Progress should not be in terms of end results but should reflect how and what goes on in a child's mind. While assessing children for counting we need to observe whether a child is properly pairing each number word with an object (one-to–one correspondence), whether her number sequence is correct, whether she understands that the last number gives the value of the whole set (cardinality), whether she is leaving out or recounting objects and so on. (See chapter on 'Addition and Subtraction'.) Only then will we be able to tell the level at which the child is and how much she has progressed.

A child of Class II, Sougata solves a subtraction problem as shown in figure 7.6.

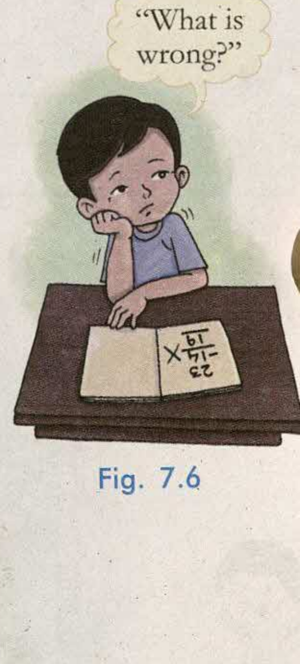


Fig. 7.6

The teacher says that Sougata has problem in subtracting two numbers and needs to practice more. Does this remark of the teacher show that she really understands what the child is doing? Is it helpful to the child in understanding why the teacher has put a cross (X)? How would you analyse the child's error?

Sougata's solution in fact shows that he needs to understand the concept of subtraction. More precisely, he needs to understand the meaning of 'decomposing a ten' and how it is used in vertical column subtraction.

From the above example what would you like to conclude about how we normally test or assess children and what needs to be changed?

The analysis of errors is a very important tool to understand what is happening in the child's mind. Even though the teacher may not have enough time to study every student's work in detail, it is very useful to be aware of the errors that children in Classes I and II commonly make.

Discuss with your colleagues and write down some common errors made by children, in two-digit subtraction.

LOOK FOR STRATEGIES USED BY CHILDREN

Children use different strategies for solving problems like adding by counting concrete objects, or counting pictures or fingers, tally marks, or by recalling from memory. Some of the strategies are taught while others are discovered by the child like while adding 5 and 6 a child says that 5 and 5 is 10 and now 1 more is eleven. Children may initially count using concrete

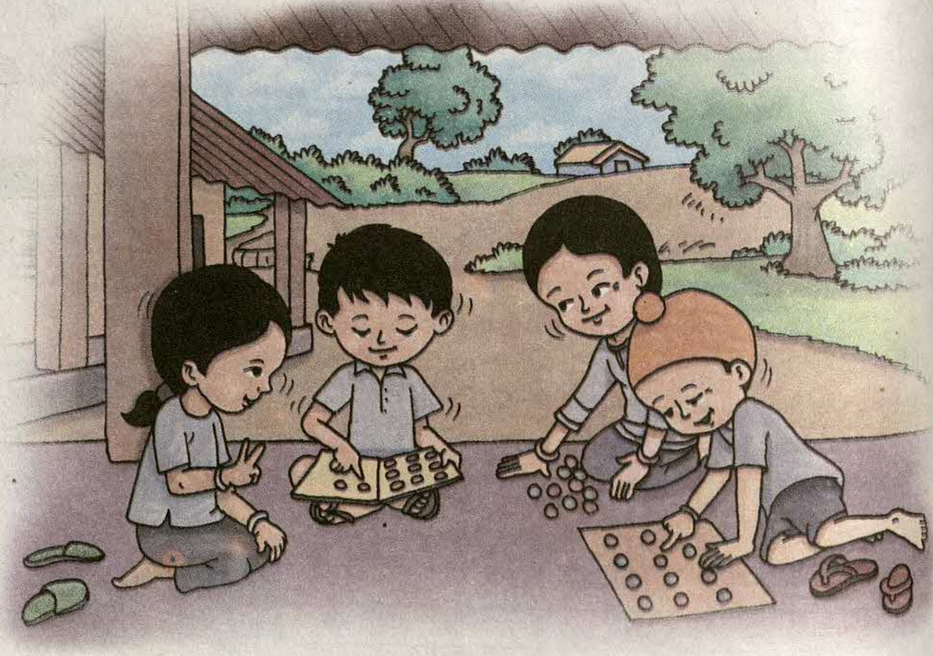


Fig. 7.7

objects and slowly move towards other strategies like applying number facts and creating their own algorithms. So, if a teacher is assessing children for say, addition or subtraction she should look for strategies a child is applying and the child's progress toward higher level strategies.

The teacher can make an assessment checklist for each concept as in the table below.

Name of the child	Can decode the numeral	Strategy used for adding					Representation o the answer	
		Concrete objects	Picture or tally	Fingers	Number fact	Derived algorithm	Pictorial	Numera
Ashraf	Yes	✓	×	×	×	×	×	✓

This kind of assessment sheet can be easily made by breaking a concept into its sub-concepts and then listing the processes for that sub-concept. In the example in the table the concept is addition. The sub-concepts are

- decoding the numeral;
- strategy for adding; and
- representation of the answer.

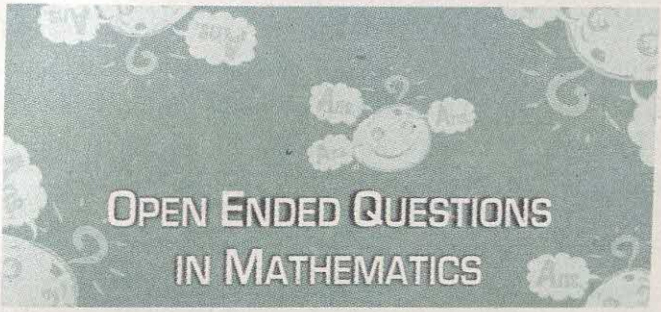
Choose examples or questions carefully

The examples or questions that the teacher uses for the purpose of assessment must be chosen carefully. To the extent possible, they should be different from the examples used in the classroom or examples that the children are familiar with. At the same time, the format of the question, and the context used in the question should be familiar to the children. The language used in the question must be simple and direct. Using pictures to aid children's thinking is helpful in making children interested and willing to think about the question.

Thus assessment is a critical part of the teacher's work and responsibility. It helps in knowing how effective the teacher's own work in the classroom is, and at the same time provides inputs for planning further teaching.

THINK AND DISCUSS

1. What according to you are the various purposes of assessment?
2. What are the different ways of assessing children's progress that you can use in your classroom? In what way would such assessment be useful?
3. What are the assessment points for assessing children's understanding of place value? Make an assessment checklist for this concept.



8

OPEN ENDED QUESTIONS IN MATHEMATICS

How did we learn mathematics in our school days? For most of us, looking back we may find that mathematics was all about solving sums and getting the correct answers. There was and is a notion that in mathematics, every question has only one correct answer.

The moment the teacher asks a question, the student becomes absorbed in the search for the right answer. In this process she tries to recall and blindly follow the methods or procedures that she was taught in order to obtain the correct answer. Even if the child succeeds in solving the problem, she has not really paid any attention to the thinking process by which solutions are obtained. This does not lead to the understanding that the problem solving activity is supposed to bring about. We need children to break out of this habit of searching for *the* single answer, or *the* correct method. We need them to think about the problem, to pay attention to processes and strategies of obtaining the solution, and to understand the mathematics underlying the problem.

One of the powerful ways to bring about this change in the students' approach to mathematics is to introduce questions which have more than one correct response, which are not product oriented, and which require thinking on the part of children. Such questions can be described as "**open ended questions**".

Why open ended questions?

These questions pose a challenge and engage the students' minds. For example, take the problem of what should be added to 32 to get 55? Here there is only one answer. Many students and teachers may insist that there is only one method—by subtracting 32 from 55. But we could modify this problem and ask—which numbers would you add to get 55?

__________ + __________ = 55

__________ + __________ = 55

__________ + __________ = 55

In how many different ways can you fill up the blanks?

The situation changes. Firstly, the child is facing a question for which there can be many answers. Different children may suggest different answers. Secondly, there is a new challenge: the child begins to look for all possible answers. Then the child faces the next challenge: finding out exactly how many answers are possible. Finding all possible correct answers is much more challenging than finding the only correct answer. Such questions not only develop the child's reasoning process, but also motivate her to look at different aspects of the same problem.

A problem of this kind gives much more scope to build the child's understanding. Note that one cannot solve this problem by mechanically following a rule, but only by understanding. The child begins to find the pattern of the numbers which add up to 55. For example,

.... 51 + 4

50 + 5

49 + 6

48 + 7.....

In doing so, she gets a better insight into the structure of addition problems. When she understands addition better, she will also have a way of verifying problems that she solves using the standard method or algorithm.

When a child learns mathematics through solving such open ended problems, she struggles with exploration and creating strategies, instead of relying on memorisation or any predetermined rules to search for the solutions. Solving a problem for which a solution is not immediately apparent gives students a chance to take a risk, which in turn can give them more confidence.

Open ended questions give a chance to every child in the class to express her views because of the possibility of multiple solutions. The child is not worried if her answer is different from another child's. Both may be correct. She has to rely more and more on her own judgement about whether the answer is right. This too helps in gaining confidence and removing the fear of maths. Hence such questions offer pupils room for their own decision making and a natural way of thinking and problem solving.

Let us look at some more examples of how open ended problems can be constructed.

Example 1

We can convert a closed answer problem into an open ended problem by making the conditions more open. Consider the problem: How many Rs 2 coins are required to make Rs 10? or How many Rs 5 coins are required to make Rs 10? These are familar questions with closed single answers, but we can modify these questions and ask-

Find a way of making Rs 10 with coins of Re 1, Rs 2 and Rs 5. Children can also use play money given in the learning kit.

Fig. 8.1

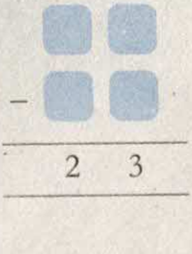
The child will look for all the possible arrangements for making Rs 10. Each child will come up with her own answer.

Example 2

We can create open ended problems by having missing data or information, which the student has to find and fill.

For example,

Which two numbers can be subtracted to get 23?



The children will search for all the possibilities and will soon realise there could be so many responses. Moreover if the restrictions like 'use only two digit or three digit number' is removed some children are likely to realise that there are an infinite number of answers.

Example 3

Instead of giving all the data for a problem, we can ask the children to make some assumptions based on their experience. *For example*: A bear weighs 110 kg. How many children are required to balance the weight?

Here the child will need to assume the weight of a child and solve the problem. Each child will solve the problem with her/his own assumption and imagination. If the question were posed in the traditional way by giving all the data: 'A bear weighs about 5 times as heavy as Chinku. If Chinku weighs 22 kg, what is the weight of the bear?', then the whole class would come with the same answer, that too, only if the children do it correctly.

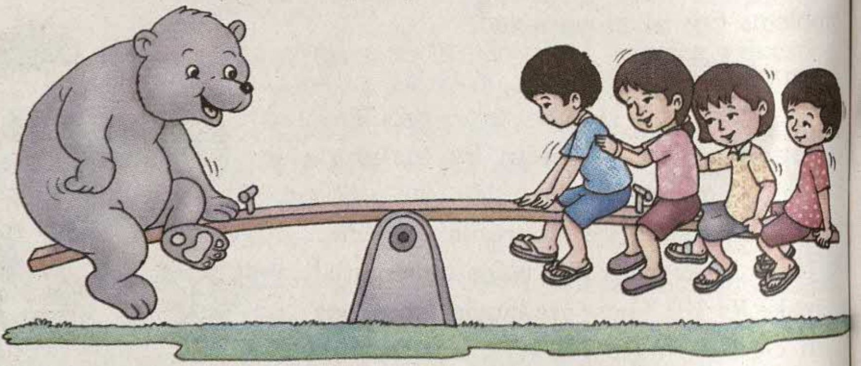
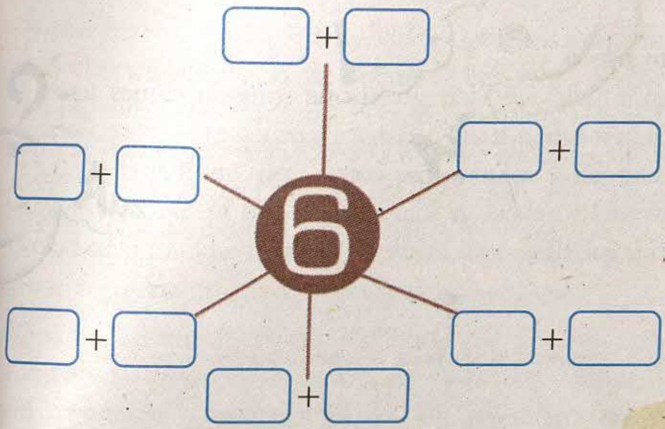


Fig. 8.2

Example 4

Open ended questions also help in starting discussions in the classroom as each student may come up with a different but correct answer. Each student may have his or her own reasoning to justify the answer and can share this with the others. During discussion, sometimes the child is able to make or formulate some rules as well. For example, if asked what two numbers can be added to get 6 as the sum, the child will look for all the possible answers and may come up with the fact that there are not more than six possibilities.

– The material, square pieces or dominoes from the learning kit can be used for this activity. The children can sit together in a group, each of them will pick up cards which will add up to the number, say 8.



This question can be extended in different directions. What about the numbers whose sum is 7 or 5? By looking at the number which is the sum, can we say how many possibilities will be there? One can also ask if there are any more possibilities. Some children may also notice that some answers are not really different. For example, 1+5 and 5+1. Do we assume that these are different or that they are the same? This can lead to children discussing the property of commutativity of addition or "turning the numbers around." (see chapter on 'Addition and Subtraction').

Fig. 8.3

Example 5

Open ended questions facilitate assessment. If a particular question has more than one correct answer, children may choose to give the answers that they are more comfortable with. For example, in the question in figure 8.4, children need to fill in 20 numbers starting from any number. Some children may start with small numbers. Others who are more confident may start with bigger numbers. This gives the teacher a way of knowing the level at which the child is comfortable.

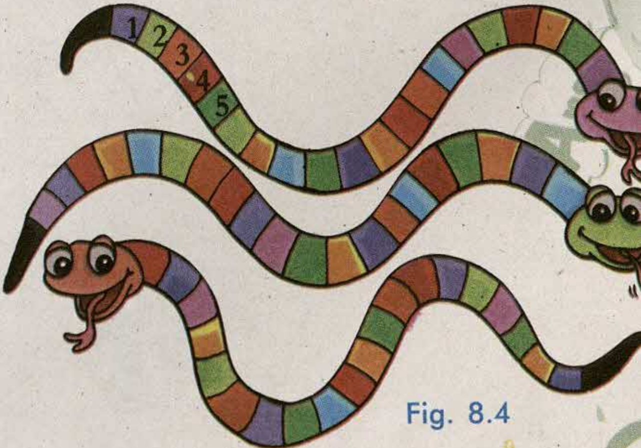
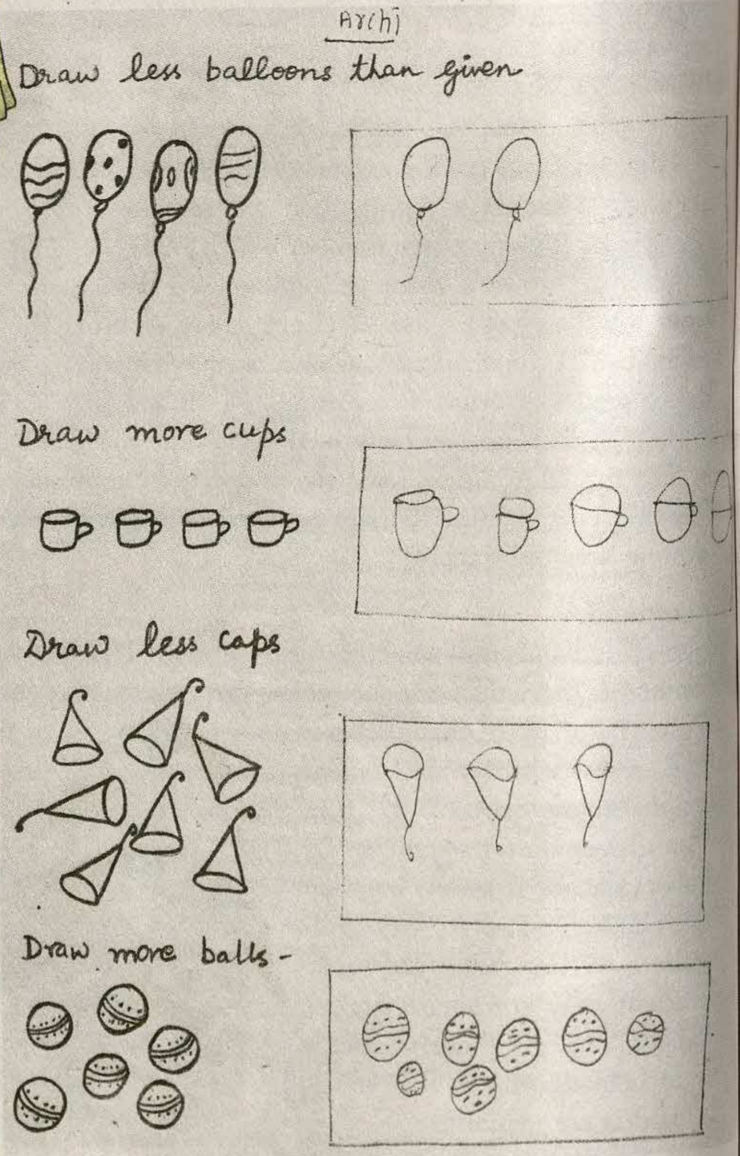


Fig. 8.4

Fig. 8.5

Example 6

When a child who has given one solution comes across a different solution, it can become a learning experience. Because the alternative solution has come from another child, she is motivated to check if it is correct, and in the process may learn something new. Look at the worksheet given below.



Archi

Draw less balloons than given

Draw more cups

Draw less caps

Draw more balls -

The teacher in Class I asked a child to draw less number of balloons than what was shown in the picture (4 balloons). So the child drew two balloons. The teacher accepted her solution and then shared with another child's drawing which had 3 balloons and asked her if it was correct. After thinking for a second, she replied yes. She was also able to explain why 1 balloon would also be correct.

How to generate open ended questions

Open ended questions can be made for almost every concept. The task is to see how typical or traditional are the questions, which generate only one response can be converted into an open ended one. Often this can be done just by changing the instruction.

Let us look at this question—Draw a joker using the given shapes.

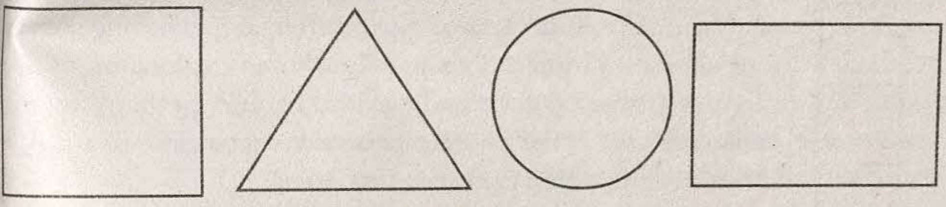
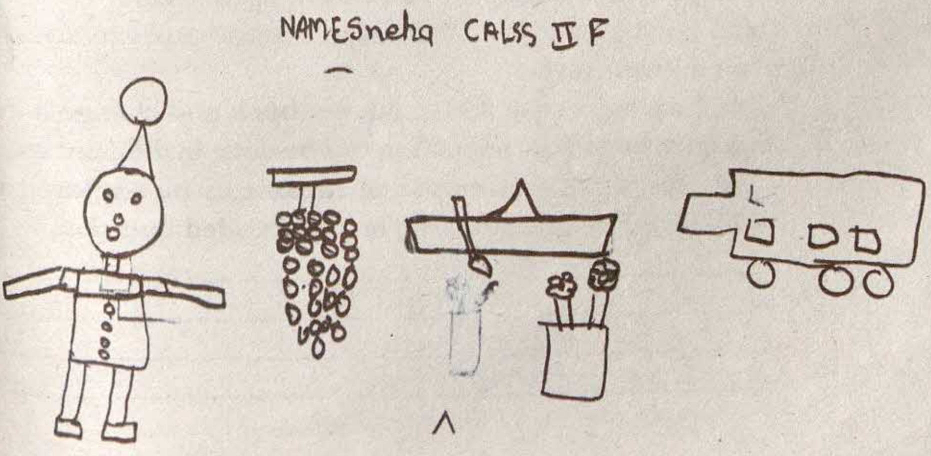
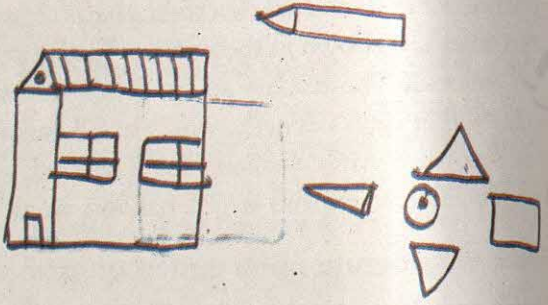


Fig. 8.6

What could be the probable responses? It is likely that most of the children will make very similar pictures of jokers. Now if we change the instructions to 'Draw a figure using the given shapes', then it is likely that the children will imagine and make different figures.





Similarly we can change the question, 'Draw a rectangle with length 5 units and breadth 4 units' to 'Draw a rectangle with sum of the four sides as 18 units'. This question is suitable for children in say class 4 or 5. To solve the first question the child will draw the rectangle with given dimensions, but for the second one the child will think of all possible rectangles having sum of four sides as 18 units. This may lead to an exploration of the relation between area and perimeter and perhaps to the realisation that if the perimeters of two rectangles are equal, it is not necessary that their areas are equal.

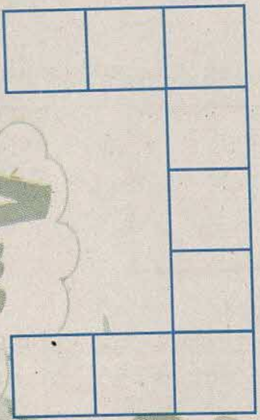


Fig. 8.7

Here is another question which can have different solutions. 'Colour one third of the figure 8.7.'

To summarise, open ended questions encourage children to explore different solutions and to think more deeply about a problem. They can share their solutions with other children and have a chance to discuss them, to justify them and to learn from others' solutions. Children also greatly enjoy exploring and finding so many solutions to a single problem and that too in mathematics.

Take a question from your textbook and change it into an open ended question. Can this be done in different ways? Discuss what mathematical ideas can be explored by changing the question into an open ended question.

9

TEACHER'S QUESTIONS AND CLASSROOM DISCUSSION

Learning occurs through various kinds of interaction in the classroom—between the teacher and the whole class, between the teacher and individual students and among the students. The questions that a teacher asks play a crucial role in shaping these various kinds of classroom interaction. The kinds of questions teachers ask can make all the difference between having an engaging and fruitful discussion or having no discussion.

Questioning is an important teaching strategy in establishing a classroom culture conducive to the development of mathematical thinking in students. Questions serve many purposes like

- catching students' attention and making them listen carefully
- getting them to express their thoughts or understanding
- provoking them to think critically and initiating a discussion.

At the same time certain kinds of questions can also create stress among the students and may lead them to accept the teacher's authority. This can close their minds making it difficult for them to learn. The students' response depends mainly on the kind of questions we ask and how we ask them. **Open-ended** questions, which have many possible correct answers, give students the opportunity to think differently and are hence less stressful.

Questions can be used to introduce a new idea and to initiate a discussion about it. Sometimes when discussions are not moving forward, the right question can shape the discussion. Questions help in probing students about what they already know as the first step towards the construction of knowledge. Sometimes students put forward new ideas and the teacher can call attention to the idea or elaborate through questions. Thus new ideas can interact with the ideas already being discussed leading to the rich understanding. Here we will see some classroom snapshots (examples) and try to understand how questioning contributes to the emergence of a learner centred pedagogy.

Types of questions teachers ask: Classroom questioning has been identified as a critical part of the teacher's work. Asking a good question is a challenging task. It is cognitively demanding. It requires teachers to know the content of the subject being taught as well as understand the learners.

Various studies have categorised the type of questions that teachers ask in the classroom. Some of these categories are summarized below. Each item lists a question type, giving a brief description and examples.

1. **Gathering information:** These questions are usually closed – they lead to a straight answer, and do not require much thinking on the part of the student. These questions may involve checking students' factual or procedural knowledge, checking for a method, leading students through a method, or rehearsing by asking students to state known facts or procedures. For example, by using different shapes, a picture is drawn on the blackboard. Teacher is asking the following questions:
   - How many triangles are there in the picture?
   - What is more – triangles or squares?
2. **Introducing or recalling terminology:** These questions are useful when ideas are under discussion, and the teacher wants students to use correct mathematical language to talk about them. For example, in Class II, the children are trying to explain why when you add 43 + 4, you cannot add 4 + 4 and get 8. The children are saying that the single digit number should be written on the right. The teacher asks, What does

'4' in '43' mean? She wants them to recall the place value terminology and to realise that adding 4 tens and 4 ones does not give 8 tens.

3. **Probing:** These questions are aimed at getting students to explain, clarify or elaborate their thinking for their own benefit and for the class. For example: When asked what is 6 + 4, child says 10. Probing questions could be like : How did you get 10? Can you explain your idea?
4. **Exploring mathematical meanings and relationships:** These questions point to the underlying mathematical relationships and meanings and make links between mathematical ideas. For example: A child is solving a subtraction problem by taking one ten from tens column. (See figure 9.1) The teacher asks: Why did we rewrite '3' as '13' and why did we change the '2' to '1'.

Fig. 9.1

5. **Linking and Applying:** These questions focus on the relationships among mathematical ideas and between mathematics and daily life or other subjects. For example: What do we say for half of half kilogram (kg)? How many quarter kg (*paov* kg) make 1 kg?
6. **Extending thinking:** These questions are aimed at extending an idea so that it can be used in another situation similar to the one under discussion. For example: If a pattern is visible in the multiplication table of three, can you see the same or similar pattern in other tables? Another example could be—If there are 6 leaves and we make pairs, nothing is left over. Does this happen for all numbers?
7. **Orienting or Focusing:** These questions help students focus on key elements or aspects of the situation to enable problem-solving. For example: In the multiplication table of 3, we find that odd and even numbers alternate: 3, 6, 9, 12.......... Is there a similar pattern in the other tables?
8. **Generating Discussion:** These questions help in starting or carrying a discussion forward. Especially they enable children other than those who are actively participating in the class to contribute and comment on the ideas under discussion. For example, Is there any other way of doing

this? Has somebody done it differently? (Don't worry about whether it is correct.) Can we do it with addition instead of subtraction?

In the primary grades, the teacher has to ask questions carefully and gently, always making sure that children feel encouraged to participate and not discouraged. Let's see some snapshots of classroom episodes where questioning plays an important part.

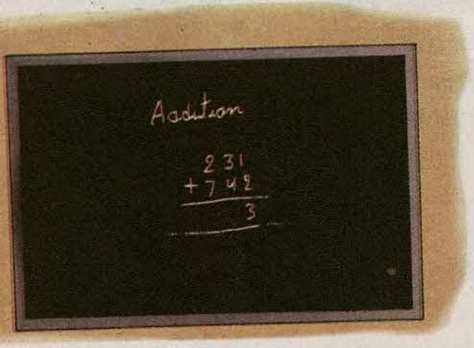


Fig. 9.2

Snapshot #1

While solving an addition problem on the blackboard a teacher added 1 and 2, and wrote '3'.

Then she asked students what to do next? Notice that by doing the first step here, the teacher assures students that the problem is easy and doable for them, so that they can proceed.

Snapshot #2

$$\begin{array}{r} 51 \\ +\ 49 \\ \hline 559 \\ \hline \end{array}$$

Teacher tries to probe the error done by Sunita.

Teacher: How many units are there in 51?

Sunita: One

Teacher: How many units are there in 49?

Sunita: nine

Teacher: How many tens are there in 49?

Sunita: Four

Teacher: How many tens are there in 51?

Sunita: Five

Teacher: In addition, units of both the numbers have to be written along one line and tens along one line. So, where will you write '9' of '49'?

Sunita: below '1'

Teacher: And '4' of '49'

Sunita: below '5'

Teacher: Write down the same way then.

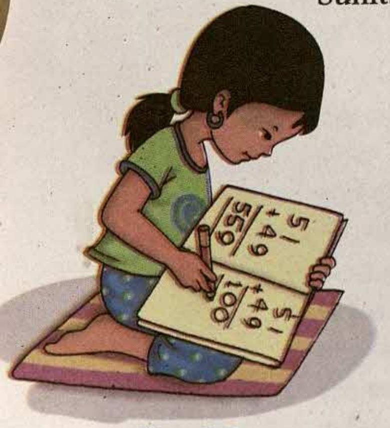


Fig. 9.3

Sunita wrote it as in the figure.

Let us look at this conversation carefully.

(i) The teacher is trying to break up the procedure into small steps. For each step, she is asking questions in a closed form to direct the child towards a correct response. The child gives the correct response, but may not understand what the connection is between the steps.

(ii) Although the child appears to know place value, she does not appear to know how it is used in vertical column addition. The teacher talked about aligning place value, but did not probe whether the student understood why such alignment is needed.

$$\begin{array}{r} {}^{1} \\ 51 \\ +\ 49 \\ \hline 100 \\ \hline \end{array}$$

How do you think that this interaction could have been different, so as to enable the child to follow the algorithm with understanding?

Snapshot #3

The teacher gave a problem to the class: Smita has 5 kites and Manoj has 4 kites. How many kites do they have together? Gurpreet solved the problem and wrote the answer as 9.

The teacher asked, "Gurpreet, what does 9 mean?"

Gurpreet: Nine kites.

Teacher: How did you get 'nine'?

Gurpreet counted using fingers for '6, 7, 8, 9'.

In this example, the teacher was able to probe the child's understanding and identify the child's addition strategy, which was 'count on'. This gave her an idea of the level of the child and what is needed to make further progress.

Fig. 9.4

Snapshot # 4

A Class IV teacher was teaching shapes and was discussing the circular or round shape. The children participated very actively and came up with names of things which were round or circular. During the discussion, one child mentioned that wheels have a circular shape. The teacher did not interrupt the discussion and it moved on to how many wheels there are in different vehicles. One child pointed out that a bicycle has two wheels: another child pointed out that some bicycles have two additional training wheels. A third child said that a rickshaw had three wheels and someone said that a car had four wheels. Then, one child stood up and said that a car had five wheels, not four.

What you think could have been done by the teacher at this point? Should the teacher have gone back to shapes and not allowed the discussion about wheels to continue?

This teacher, in fact, did not control the discussion and allowed another child to ask where the fifth wheel in the car is. To this the child replied that the fifth wheel is the spare wheel that is kept in the dickey of the car. Suddenly, this answer triggered many other interesting observations by children on the number of wheels in the vehicles. One child said that if we are counting the spare wheel then we should also count the steering wheel. The discussion moved further on to which vehicle has the maximum number of wheels. At about this point the discussion came to a halt. The teacher then asked a question to move the discussion further. She asked why the wheel of a vehicle was circular and not any other shape. To this the children responded that it is circular to make it smooth so that it can move easily. One child stood up and said that we would get four bumps if wheel had a square shape.

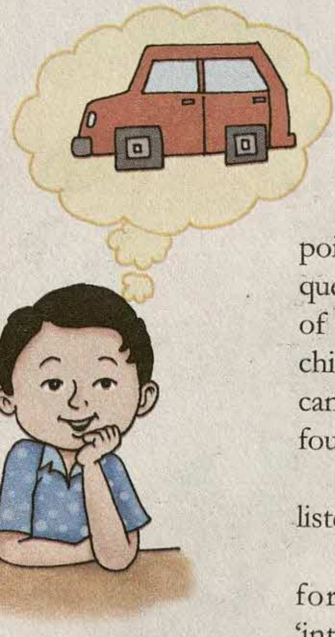


Fig. 9.5

What do we see in this discussion? Were the children listening and responding to each other?

Can you see how each new idea moves the discussion forward leading to other ideas. This process is called 'interanimation of ideas'.

Did the children learn anything important about the properties of a circle?

Snapshot # 5

A Class IV teacher was showing blocks for ones, tens, hundreds and thousands in her class.

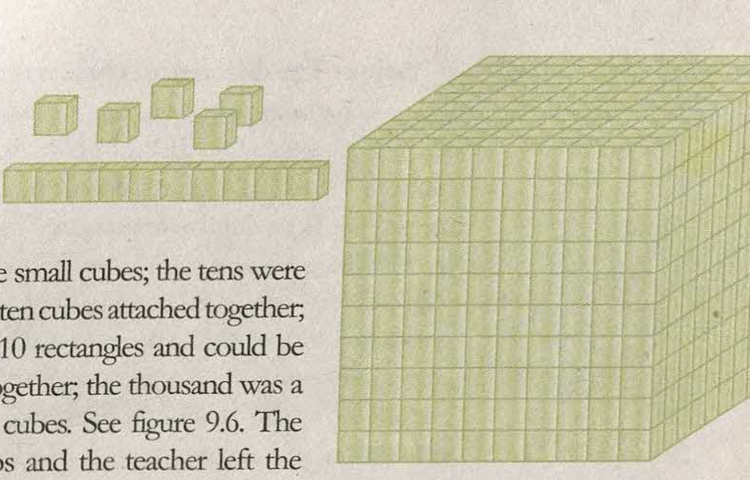


Fig. 9.6

The blocks for ones were small cubes; the tens were represented by a long stack of ten cubes attached together; the hundreds were flat 10 x 10 rectangles and could be got by attaching ten 'longs' together; the thousand was a cubical block of 1000 small cubes. See figure 9.6. The class was divided into groups and the teacher left the blocks with the students and asked them to observe them first. After 5-8 minutes she came across a discussion between two students about how many small cubes there are in the big cube (thousand box). Nikita was saying there are 500 and Shabana said there are 1000 small cubes in it. Let us see how teacher handles it.

Teacher: There are two people in the class who are thinking differently and they will tell us about what they think, let us listen to them.

Nikita: The block of a thousand is not actually thousand cubes but it is made of 500 cubes, as each wall has 100 cubes.

Shabana: Nikita is saying there are 500 cubes, she is only counting the visible cubes.

She keeps rectangle of 100 on top of each other. After putting ten such she says, see now there are 1000 small cubes in this big box.

(Meanwhile Nikita realises that her answer should be 6 flats and hence 600)

Teacher: Why 600?

Nikita: I did not count the bottom, if I do so, then I have 6 walls, therefore 600 cubes.

Shabana: Nikita is going around and counting only the cubes on the sides.

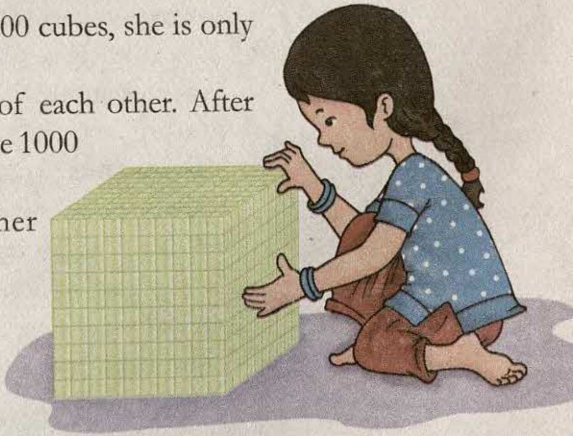


Fig. 9.7

Aliyah: I have made both blocks—the first a hollow block using six flats and the second using 10 flats.

Teacher: Somebody wants to explain differently?

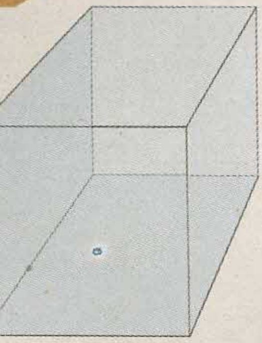


Fig. 9.8

Arjun: The difference is Nikita is counting only from outside and Shabana is counting from inside and outside both.

If the teacher had just told Nikita that, no Nikita, there are 10 such blocks of 100 in thousand box, the students would have missed the clear demonstration by Shabana of how ten flats could be stacked to make a thousand block. Also the teacher moved the authority of decision making to students, which resulted in a good conclusion, that one can either count only from outside or one can count from inside as well as from outside. This example shows how the teacher's presence of mind can bring good discussion and interanimation of ideas in the classroom.

Conducive Environment for Discussion

For initiating a fruitful discussion in the classroom, a conducive environment is required. Some features of a conducive environment are the following.

(a) It is very important to generate a feeling that in the classroom, all the views, and ideas are listened to attentively by the teacher and other students and responded to. If it happens, slowly the children will start expressing themselves without any inhibition, and very lively and meaningful discussion can take place in classroom.

(b) The role of a teacher is of a facilitator, who, at times, provides input that fosters and promotes deeper, focused thinking, which in turn enhances the level of the discussion. The students should not think of the teacher as an authority figure, who everytime will give them the correct answer. They will learn to become self reliant.

(c) Active participation of all students including those who are shy or hesitant has to be encouraged in order to have a fruitful discussion in the classroom.

THINK AND DISCUSS

In the last two snapshots we have focused on discussions and how the teacher's questions can shape the discussion. How would you categorise the questions asked by the teacher in these snapshots? Think about what role these questions play in the children's learning.

10

Managing Group Work

In a typical classroom some children are shy, some are very active, and some are inbetween in communicating their thoughts and participating in classroom activities. Some are good at solving problems while some are good at reading and writing. Sometimes the task that the teacher assigns is very easy for a few children as they have already mastered the task, while some find it challenging. In other words, there are individual differences or variability among students in our classrooms.

Let us look at an actual classroom situation faced by a Class I teacher and see how she deals with the situation:

From the children's performance in a mid-term maths test, the teacher found that there were 8 children out of 45 in her class who could not read numerals beyond 4 or 5, while 12 could only read numerals up to 10. 18 students could only recognise numbers up to 20. There were 7 children who could read numbers up to 35 or 40. Looking at this variation the teacher wondered how she should plan her teaching.

Some students were ready to progress up to 50 and beyond, while others were struggling with counting. She decided to take up activities in her class, and have the students work in groups where they could help each other. Those who had mastered numbers up to 35 should not get bored and those who only knew numbers up to 4 or 5 should not feel threatened. She formed groups of 3 or 4 children with at least one in each group who could read and write numbers up to 20 and planned to have activities to help children understand numbers and learn numerals up to 20.

One of the activities was making a set of number cards from 1 to 20 for each student in a group. Each child had to make his or her own number card, but children in a group could help each other in writing numerals and drawing the correct number of objects/ pictures at the back of the card. The teacher gave them blank cards. She asked them to first write a number 1 and then make a picture at the back. She gave similar instructions for each number card till 5 and waited till they had completed the cards. Then she asked them to make number cards up to 20. She asked the children to help each other in writing the numerals and to check each other's cards to see if they made the correct number of pictures. She observed that everyone was talking to the others. Some children were correcting others in the group. This activity took more than an hour to complete but at the end everyone was ready with a set of number cards.

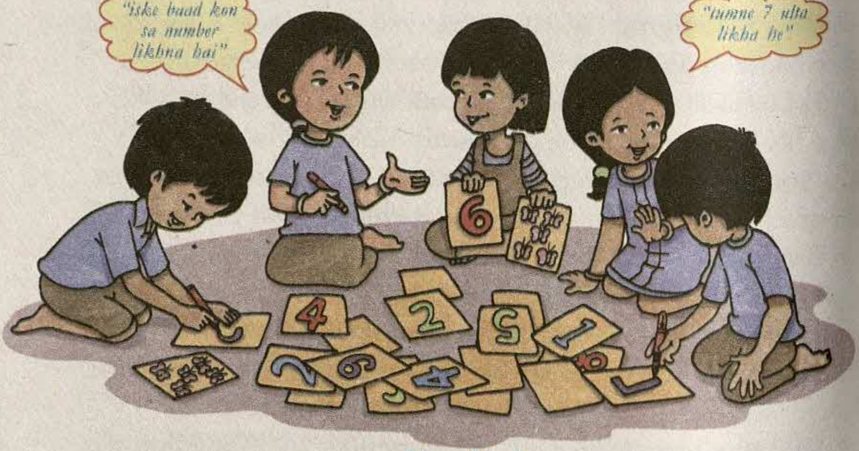

"iske baad kon sa number likhna hai"
"tumne 7 ulta likha he"

Fig. 10.1

After this she introduced a few more activities where children could use the cards. One such activity was playing 'Nambola', where each child was given a set of number cards from 1 to 10. One child in the group who could read and write numerals was made the 'number caller'. He would pick out a card and read the number aloud without showing the card to the others. The others had to locate the card from their set of cards and write the numeral in their notebooks. The child who took out the correct card first was the winner of the turn. After 5 turns, another child would be the 'number caller'. In this activity too, children were talking to each other and discussing if their card was correct. The teacher observed that some conflicts arose within the groups about who will call the numbers first or who the winner was. She needed to manage those conflicts so that the activity went smoothly. Some children came to her to confirm the number name for a card. Some children were peeping to see what the other groups were doing. She found that everyone was busy in handling, talking and playing with the numbers.

When she reflected back she found that there are many advantages of the group task but at the same time there were also many challenges.

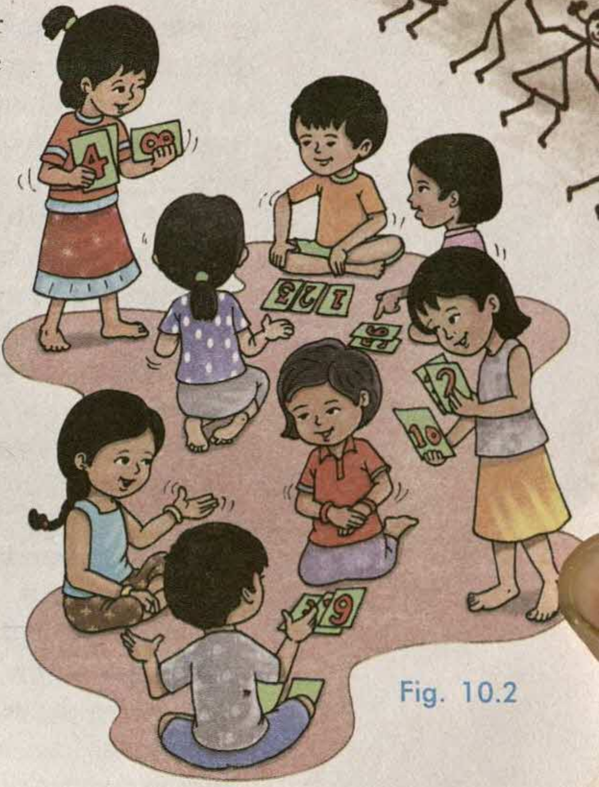


Fig. 10.2

Is the teacher able to observe and give feedback to most of the children on every number?

Advantages of group task

- **Communication among peers:** It increases in a group setting. They sit with each other in a group. They talk to each other not only about their school activities but also about other things. They create their own social space in the classroom, where they share many things in the group. They help and support each other in their learning and thus improve the learning environment in the classroom.
- **Mutual influence:** Support from teachers and peers encourage shy and average-performing children to become

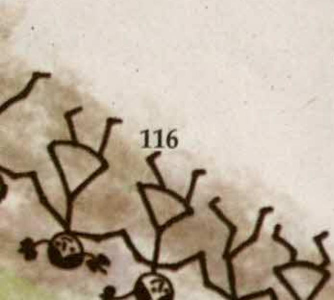
involved academically in the group. The better performing children can help others in learning the concept. While doing so, they are also deepening their own understanding of the concept and improving their reasoning and communication skills.

- **Taking responsibility:** Group tasks provide children an opportunity to take responsibility by adopting and even forming their own group norms. Children who seek attention, those who often disrupt the lesson, those who learn fast can all assume appropriate roles in the group, such as being leaders or facilitators. Group work can thus reduce problems of indiscipline and facilitate classroom management.
- **Better learning:** Some research studies also support the use of group work as leading to more effective learning in many contexts. Children in groups have opportunities to actively work on the concept, see others do the task, talk to each other on the same concept, and be more involved in performing the task. Thus, with well-designed group tasks, they tend to learn what is taught better and retain it for longer than in the case of individual learning.

But as seen in the example discussed, arranging the class and creating the group learning environment takes some effort on the part of both the teacher and the students. It is not easy to make everyone work on some clearly defined but flexible tasks. There could be many challenges that a teacher needs to handle such as:

- Making the classroom arrangement suitable for group work.
- Adjusting the teacher's role in the group work.
- Designing appropriate activities.
- Giving clear instructions.
- Managing conflict among students in the group including disagreement on role assignment, confusion regarding sharing of materials, etc.
- Managing the noise level and overall atmosphere in the class

There is no readymade solution to these challenges. As teachers we need to search for the solution that is appropriate to our situation. Some of the issues are discussed here.



HOW SHOULD A TEACHER FORM A GROUP?

There are no fixed answers to this question. It depends on the situation, the task chosen and the goal of the activity. Broadly speaking there are two possibilities from which the teacher can choose.

Possibility I: A teacher can form a group of those children who have already mastered the concept and can give them some challenging tasks. Similarly, she can put those who need extra help together in another group. This is called 'same ability grouping'.

Possibility II: She can form 'mixed ability groups' where students who have mastered the concept can help those who have not understood the concept.

It is found that in most situations mixed ability grouping is more suitable than same ability grouping. If a group only has students who cannot read, then they will be lost when an adult is not there to support them. Frequently this leads them to becoming distracted leading to disruption of the class. In a mixed ability grouping, children often learn from each other. They share their thoughts and strategies and support each other in overcoming their weak areas.

Points to be taken care of while working in groups

How efficiently a group functions depends upon various factors such as:

- Group Members
- Group norms
- Type of tasks allotted in a group
- Instructions given for the task in each group
- Students' participation in the group task
- Presenting the group work in front of the whole class
- Feedback given to the group as well as individual student

Group Members

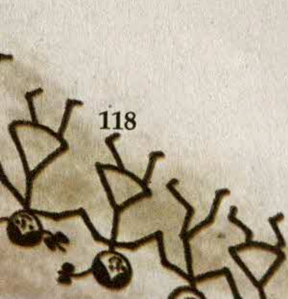
(a) The number of children in a group should be decided by considering the following points:
  (i) Tasks to be done in the group
  (ii) Materials to be shared
  (iii) Compatibility of children to work together in a group.

In various research studies it is found that the optimum number of children in a group task is 4 to 5. However, this will also depend on the demands made by the activity and the number of children present in the class. Also, as the children of Class I are very young and may be very new to formal school settings, it may be difficult for them to work with 4 or more children together. It might be preferable to have them work and play in small groups of 2 or 3. Gradually when they start talking to each other and are able to share their thoughts, then the number of children in the group can be increased.

(b) The teacher also needs to take into account the children's likes and dislikes. If children are not comfortable with each other then it is possible that they may not interact with each other leading to the objectives of the group work not being met. For example, if two students who have quarreled frequently are put in the same group, they may avoid each other and this affects the group interaction. Thus the teacher needs to talk to the children while forming the groups. She needs to be open to shuffling the group members if there is any problem. However it is not a good idea to change the groups frequently, or after they have settled down.

Group Norms

To ensure that group work proceeds smoothly, some group norms (rules) and arrangements need to be made before starting the tasks. Group norms can be made after having discussion with the children, so that norms are acceptable to all and are not forced. If rules or norms are not put down, conflicts might arise delaying or preventing the group task from being taken up. For example, if the group is asked to take out beads from a box, there may be a conflict over. Also who will take out the beads first. So the teacher needs to establish some rules or norms for every group task about who will be first, how to take turns, what the role of each student in the activity is, etc., before actually starting the activity. This needs to be done especially when students are new to group work. Children take time in developing group norms. Hence the teacher should explicitly



specify norms along with assigning the activity. Gradually the norms become a habit among the children.

Besides setting up the group norms and rules, the teacher may need to modify the classroom arrangement to make it suitable for interaction among children. For example, chairs or desks may need to be moved so that the students can talk to each other.

Fig. 10.3

Type of Tasks

Generally, the aim of the group work is to complete a task or activity with a collaborative effort. In the initial school years (Classes I and II), it is difficult for young children to think, interact and convince other group members easily. Collaborating with each other is a higher order skill which needs to be developed in children by first helping them to work in groups. Thus, initially the tasks in groups should be simple requiring only simple group roles from the children such as sharing of material, helping each other in completing the task, ensuring that every one in the group has done his/ her task at hand, etc.

The following four types of tasks can be given to a group according to the situation and the needs of the group:

1. **Working individually in a group**: Children sit in a group, share their working space, materials, etc. but are assigned a task to be done individually. They can share their work, strategies, explain instructions to each other, etc. However, the product of the activity will be individual, for example, a worksheet on comparing numbers to be completed by each child sitting in groups of four children.
2. **Working competitively in a group**: Children sit in a group, share the learning material but the task provides a situation where children compete with each other in a small group.

Fig. 10.4

Fig. 10.5

For example, while playing snakes and ladders game on the square board. Here the child who reaches 100 first wins the game. Similarly, children compete in small groups while playing with dominoes. Like, in one of the games played with dominoes, one child says a number say '6'. Rest of them have to pick up all the possible addition facts from their respective set of dominoes. For instance, the child who picks up all, at the earliest is the winner.

3. **Working cooperatively in a group:** The task is such that it can be done only if the children support each other. For example, in the activity of making number cards, although each child had to make a set of cards, the group as a whole was responsible for making four sets of cards (if there were four children in the group). So they were helping each other in counting, writing numerals, checking each other's cards, etc. Children were simultaneously helping others while working on their own task.
4. **Working collaboratively in a group:** Here the children have to think, plan, execute and present the task in a group with the consensus of all the members in the group. To take an example done with slightly older children from Classes IV and V, the teacher set up an activity of making their own scale for measuring length by choosing a familiar length as unit, such as the

Fig. 10.6

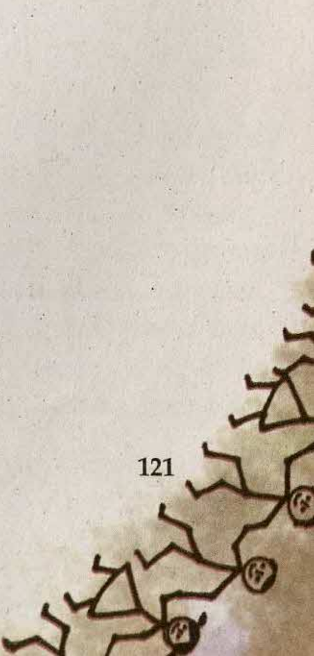
length of their textbook. The children used this scale to measure the length and breadth of the table and of other objects. This activity required much discussion and collaboration. The children had to think and decide which type of scale they would make. They had to decide who would make the scale, draw the marks on the scale, who would color the scale and who would measure the lengths of the objects. All these steps also had to be done by two or more children working together and helping each other. And after measuring and noting down the measurement of the objects in their copies, they had to decide who would present the task to the other groups and how the data they had collected would be presented. They also had to listen to the other groups, check their measurements and give feedback to them. This was a demanding but enjoyable activity for the students.

Role of Instructions

Clear instructions from the teacher are very important in group work. Firstly, the teacher needs to ensure that all children have received the instructions. The teacher may give the instructions first to the whole class and then check if students have understood the task by talking to them in the group. The teacher should pay attention to the language of the instruction which should be simple and clear. Instructions may need to be broken into important steps. Each step needs to be explained clearly with examples so that there is no confusion. The teacher can ask questions to confirm the students' understanding of the task needs.

Students' Active Participation in the Group

It is important to make sure that every one is actively participating in the group task. There is a tendency among some students to become leaders of the group and to make all the decisions for the group. This need to be taken care of as it might affect other students' participation and interest in the activity. A teacher needs to be observant and thoughtful in her response. For example, if she finds that a student is dominating, she can modify her role intelligently by asking her to help others, distribute the material or ask questions to the other group members so that they can also start participating in the group.



Feedback and Presenting Group Task to the Whole Class

The group task need not end with the students completing the task and the teacher giving feedback. Students like to share their work with their classmates or friends. The teacher needs to provide opportunities for this, for example, by having the students put up the work done on the wall/board and present the task in front of the whole class. Students can then get to know what other classmates are doing and get feedback from them as well. In this way they not only learn from others but also feel confident in presenting their tasks.

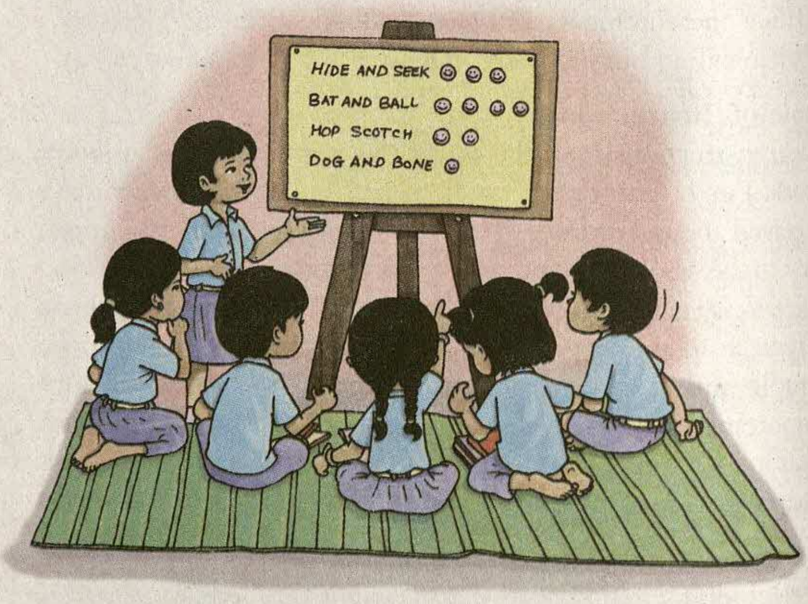
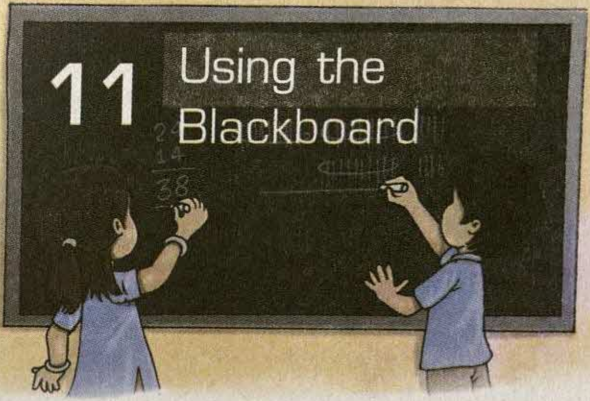


Fig. 10.7

To summarise, the teacher's role in managing group work is critical. She has to play a mediating role, helping each group to achieve the objectives of the task. The teacher has to be flexible and democratic. If the group work is managed well, it can become a powerful tool for learning.



11

Using the Blackboard

It is difficult to imagine teaching a lesson without a blackboard or chalkboard. But there are many classrooms where this essential aid to teaching is in poor condition. It is extremely important that every classroom has a blackboard in good working condition. As all teachers know, this can be a very effective visual medium of communication. Here we will discuss some ways of making the blackboard a more effective medium.

What does the blackboard do? It projects the teacher's thoughts to the whole class. It can also function as a record of what was done in the lesson. It is not only teachers who can use the blackboard in this way, but also students. It is a common social space and a platform for sharing something important. In many schools there are blackboards running all along the wall for children to write on. The height of these blackboards is suitably low for children to use. Children utilise this space with complete freedom. They can practice, scribble or do whatever they feel like.

There are some simple and commonly known rules about the use of the blackboard, which are worth recalling. Firstly, the blackboard surface should be maintained by regular cleaning, painting or polishing. If there are administrative hurdles in getting this done, the teacher can get it done with the help of the community. The teacher can also explore alternatives. The black plastic sheet, which is used in small houses for roofing or waterproofing, can be used as a blackboard. Using white paint one can even paint a line grid or a dot grid on the plastic sheet, and then use it as a square-lined blackboard, which is useful in drawing figures.

Second, the blackboard should be placed where the light illuminates the blackboard, but the reflection does not disturb what can be seen. Adjust the children's seating so that all the children in the class can easily see what is on the blackboard. If any of the children have an eyesight problem, seat them closer to the blackboard. (Also ensure that they see a doctor and get glasses if needed.) The writing on the blackboard should be large enough and clear enough for the students to read easily. Many teachers prefer to use 'print writing' (separating the letters in a word), rather than cursive writing, because it is easier for children to read.

As teachers, we also need to pay attention to the organisation of the material on the blackboard. Look at the examples given in the figures below.

Which blackboard do you think is more readable and helpful for the child?

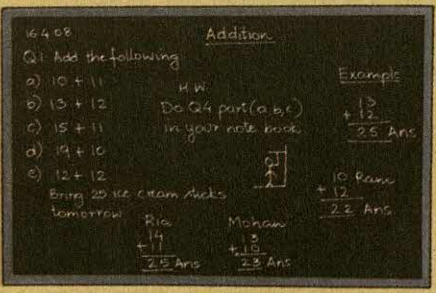


Fig. 11.1

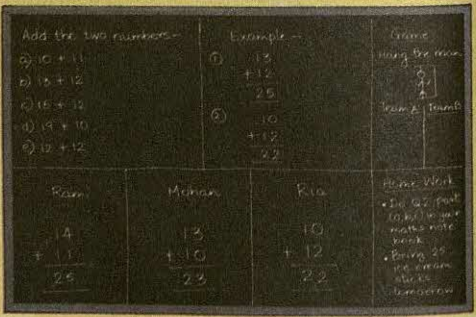


Fig. 11.2

If we look at the layout on the second blackboard, we can see that there is a separate well-defined space for homework or reminders. Lines separating different tasks ensure that tasks do not get mixed up. If the teacher is consistent with her layout then children become used to it and follow it easily.

BLACKBOARD AS A VISUAL RECORD

One of the most important principles in using the blackboard is to make the visual material on the blackboard as self-sufficient as possible. Teachers usually write on the blackboard and also talk at the same time. But the children may not pay attention to what the teacher says or may forget what she has said. The writing and drawing on the blackboard remain after the speech is gone. So when the children look at the blackboard they should, as far as possible, be able to make sense of what is there without the help of the teacher's speech. We discuss this with an example, presenting the division algorithm.

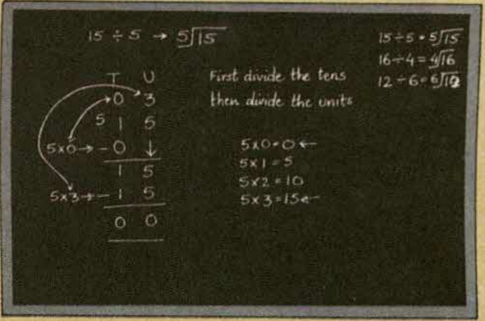


Fig. 11.3

When the teacher introduces a new concept and explains it to the class children might miss segments of oral speech. If the teacher supports her oral speech with visual pointers on the blackboard then it helps the children understand what the teacher has said. In figure 11.3 you can see the division algorithm shown on the blackboard.

The division algorithm is a difficult algorithm involving many steps, with complex rules at each step. If the complete algorithm is shown on the board, the in-between steps get hidden. So the teacher has tried to illustrate the different steps on the blackboard visually. See the figure and identify where the teacher has shown the various steps of the division algorithm such as converting the division question into an arrangement for starting the division algorithm, making the place value columns, identifying the correct digit in the quotient, etc.

With this presentation on the blackboard even if the child misses out significant things, she can get back to them with the help of the blackboard. The presentation of the algorithm can remain there till the end of the class. The conventions and visual pointers will help children in forming the relationship between the known (÷) and the unknown (⟌) symbols and in identifying the different steps.

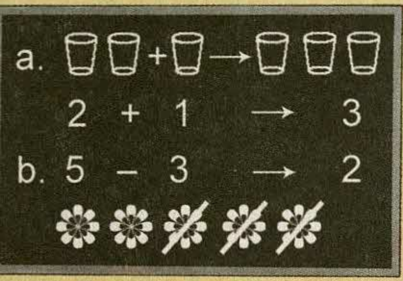


Fig. 11.4

Visual presentations need not be static. They can also show a process with the use of various conventions. Look at the examples in figure 11.4 which show the usual symbols of addition and subtraction.

Are these symbols static or dynamic? Do they convey a process?

____

____

The blackboard is a space where you can put in lots of information which can stay with child for the whole period or even for the whole day. If we think carefully before the lesson and plan what things need to stay for more time on the blackboard and what things can be erased, it can make a lot of difference to the lesson. The blackboard is therefore like a visual recorder, which can keep the record of whatever is happening in the class. It is up to the teacher how she uses this recording device.

Blackboard as a Visual Aid

Many children find it difficult to read and understand word problems. In such a situation it is helpful to add visual support to problem solving. Using simple line or stick figures the teacher can present a situation and ask students to look at the figure and interpret what is happening. Many children can describe the picture and also formulate 'story problems' about the picture. This is a first step towards solving the problems.

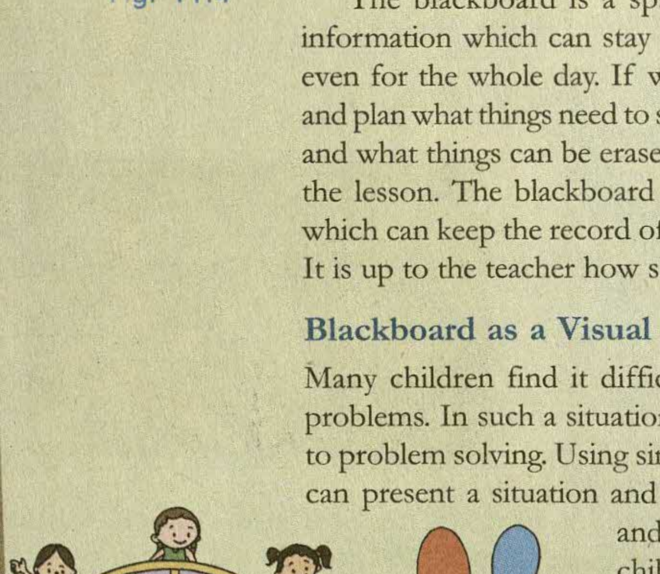


Fig. 11.5

Drawings made on the board can act as substitutes or as visual support for word problems. They make the problem interesting and engage the attention and thought process of the child. They can also be used as a starting point for the task of describing a situation in words, which children find difficult if they are not using their mother tongue. It is not necessary to draw perfect pictures – one can use simplified line drawings. With a little practice, teachers can learn some simple illustrations. A very good reference is the book written by Eleanor Watts, '*The blackboard book*'. Figure 11.6 shows some of such drawings.

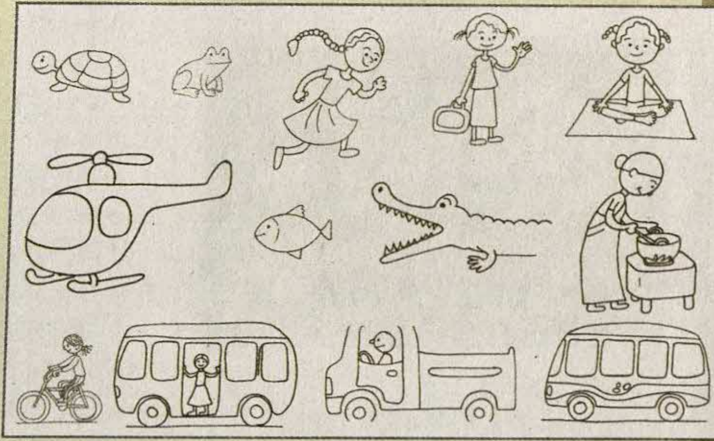
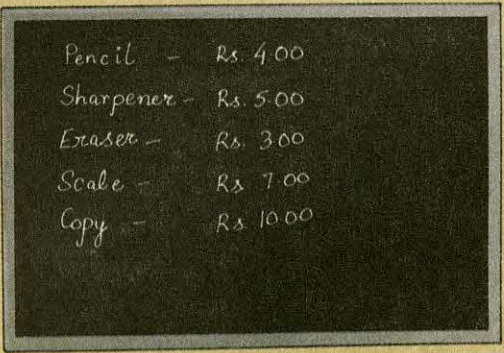


Fig. 11.6

When the teacher is making a drawing on the blackboard, it may take some time. How does one keep the children engaged during this time? One solution is to prepare the blackboard before class or when the students are busy with some other task. Another solution is to engage the children with what is emerging slowly on the blackboard by talking to them as you draw or write on the blackboard. One can ask engaging questions like, what am I drawing? What will I draw next? Am I drawing a horse or a camel? If you are giving them sums on the board then you could ask questions like, how many questions do you think I will give you today? Are these problems on addition or subtraction? Which number I am going to write next, will it be greater than the previous one or smaller? Such questions engage the children in a meaningful manner.

Another important aspect of blackboard is that it can be used to record and display data. For example, if children are playing a game involving groups who are buying and selling articles, then both the buyers and sellers need to know the price list, which can be put up on the blackboard.



Pencil	-	Rs. 4.00
Sharpener	-	Rs. 5.00
Eraser	-	Rs. 3.00
Scale	-	Rs 7.00
Copy	-	Rs 10.00

Fig. 11.7

For many of the activities described in the chapter on

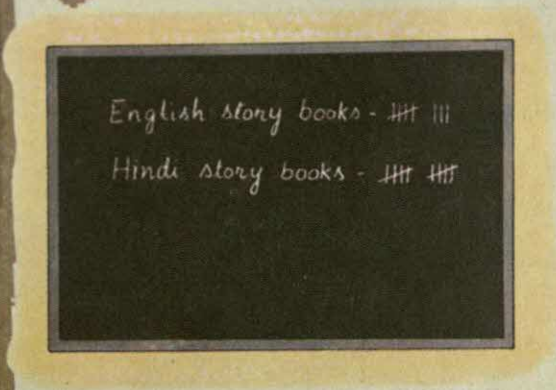


Fig. 11.8

'Collecting and Using Data', there is a need to display data in front of the whole class. If this is done, children can see how the data is recorded and organised, and dicuss and interpret the data. We can also ask students to put the data on the blackboard themselves, which is interesting as well as challenging for them. The data displayed on the blackboard in figure 11.8 shows how many children like to read story books in English and how many in Hindi.

The data can remain on the blackboard till children have discussed it and have compared and analysed the data.

Preparation of the Blackboard

It is very helpful to the teacher to do some planning and preparation of blackboard work before the class. It helps the teacher in visualising how to make the best use of the blackboard space. In Japanese classrooms, teachers plan their blackboard work very carefully. What will be written, what will stay on the blackboard till the end of the lesson or the segment, and what will be erased is planned before hand. Such blackboard planning is similar to the lesson planning and includes planning how the blackboard will look at the end of the lesson. The example in figure 11.9 shows the blackboard work that was planned for a problem solving session in the classroom.

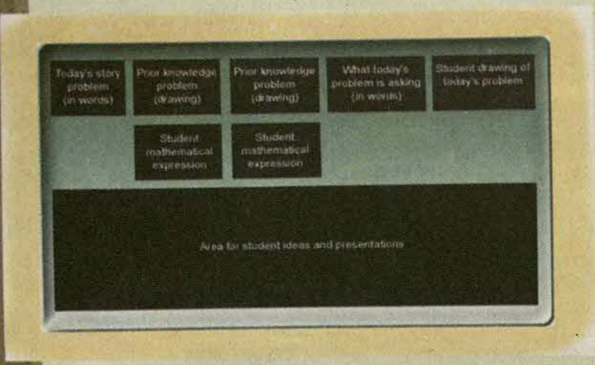

Today's story problem (in words)
Prior knowledge problem (drawing)
Prior knowledge problem (drawing)
What today's problem is asking (in words)
Student drawing of today's problem
Student mathematical expression
Student mathematical expression
Area for student ideas and presentations

Fig. 11.9

Look at the Figure 11.9 carefully. What are the aspects of the lesson that the teacher has paid attention to ? How has she tried to incorporate students' responses and prior knowledge?

Figure 11.10 shows the actual use of the black board in a Japanese classroom. According to Makoto Yoshida, a Japanese mathematics educator, blackboard work helps students to (i) see a record of the lesson (ii) remember what they need to do and to think about (iii) see the connection between different parts of the lesson and the progression of the lesson (iv) compare, contrast, and discuss various ideas that have been presented (v) organise thinking and discovery of new ideas (vi) developed organised note-taking skills by presenting a model of good organisation.

Use of the Blackboard by the Children

The blackboard offers a huge space for the child; it is like a big canvas to express, present and share thoughts, ideas and feelings. It is no wonder that children have a fascination for the blackboard and try to write on it during the break. However, in many classrooms, the blackboard is a writing space exclusively for the teacher and the notebook for the children. Instead of the blackboard remaining distant from children, one can use its power to get children to express themselves and to communicate something to their classmates. Many adults believe that children only scribble and will not be able to 'properly' use the blackboard. But if we offer them meaningful opportunities to use the blackboard, it can become more powerful as an aid to learning.

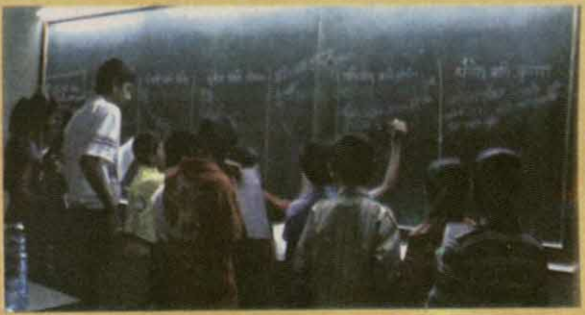


Fig. 11.10

When children complete a group task, the blackboard is a convenient space to present and share their results. The presentation motivates the students to engage in the task and also provides an opportunity to see what their friends have done. In the example in figure 11.10, children of Class V are presenting their solutions. Each group drew a picture to represent the problem and then presented their solutions. Sometimes the students took help from the other groups by observing what they had written. Sometimes they found mistakes in what the other groups had written, and then found mistakes in their own writing; sometimes they found good ideas that they could implement. This was a good learning experience.

Calling Children to the Blackboard

Calling children to the blackboard is a helpful way of engaging the students and motivating them. For the teacher, this is a way of getting feedback about the students' learning. For the students, it provides an opportunity to learn from others, to present new ideas, alternative solutions, etc.

When something is presented on the board, children can themselves identify the errors that are there and can correct themselves, or take help from their friends in correcting their solutions. The rest of the class usually promptly identifies and points out the errors. Errors made by children give insights to the teacher about the gaps in the understanding.

There are several challenges for the teacher in managing this activity. Firstly, the student who presents a solution on the board and finds that it is wrong may be discouraged. This may happen especially if the class shouts and points out the error! One strategy to avoid this is to call three or four students together to the board and let them present solutions to the same problem on different parts of the board. Here they can learn from each other, and if they all make mistakes then it becomes valuable feedback to the teacher.

Another challenge in calling students to the blackboard is how to engage the rest of the class while one child is working on the board. One can adopt various strategies to overcome this difficulty. For example, divide the class into say four groups. Divide the blackboard into four columns; four children, one from each group will come to the board and do either the same problem or different problems, while the rest of the team members solve those problems in their notebooks. There can be points for correct solutions, for good solutions, or for solutions presented well. Other students who are sitting in their places can check the solutions of their group or of the other group.

In general, when a child makes an error, the teacher has to handle it sensitively. The child should not feel discouraged. Also if the teacher begins to explain the point to the child at the blackboard, the rest of the class may be distracted and feel left out. The teacher can avoid this by explaining the point to the whole class or by waiting till she gets time with the particular individual student.

SHARING OF IDEAS

Many times there are students who solve the problem in a different way. Such alternatives are meant to be shared with the whole class. Therefore one can give opportunity to children to come up to the blackboard and explain to the whole class how she solved a particular problem. When children give explanations and rest of the class responds, a discussion can be generated, which helps in forming a better understanding.

These are a the few suggestions, teachers can think of many more ways to use the blackboard. If both students and teachers use the blackboard optimally, then it can become a truly powerful tool for teaching and learning.

Analysing Textbooks

12

Fig. 12.1

NATIONAL CURRICULUM FRAMEWORK 2005
Syllabus for Classes at the Elementary Level

Fig. 12.2

Children need print material for reading and learning. They need material, which they can own, which they can read and write on, practice, etc. For many children, textbooks are the only print material available. Parents also feel informed and satisfied when their children have textbooks. Textbooks are available to almost every student. In most school systems, the government subsidises the textbook, and in many cases makes it available free of cost to students who need support. Hence textbooks play an important role in the education system.

The textbook is based on the curriculum that is adopted in the school system. In the formal education system, which has the goal of providing good quality education to all children, what is to be taught is carefully planned and is decided at the national or state level. Curriculum framers discuss why a certain topic is to be taught and what approaches are to be adopted. The *National Curriculum Framework (NCF)*–2005 is one such document. It is based on a variety of considerations like philosophical ideas and educational policy and how students learn. Based on all this, it formulates the aims and guidelines for school education. On the basis of the curriculum framework the Syllabus for different stages are formulated and give th outline of the concepts to be taught in an academic year. Th textbook authors consult both the curriculum framework and the syllabus document in detail while writing the textbook.

Thus, the textbook is an interpretation of the curriculum and the syllabus by the authors who write it. But most important, it is the teacher who transacts all this and actually meets the learning needs of the children. The teacher needs to understand the overall curriculum goals and to work towards these goals in the classroom. To effectively manage the learning of the children in accordance with the curriculum goals, the teacher needs to be familiar with the curriculum documents at different levels. The curriculum framework and syllabus documents are generally publicly available, either in the school or on the website of the concerned Government body like the NCERT.

www.ncert.nic.in
www.scertdelhi.org

Of all these documents, however, the textbook is the most important from the point of view of the teacher. The teacher can gain a lot from a careful study of the textbook. Such a study helps the teacher to understand the structure of the topics to be taught, the concepts, the points which have been emphasised, and the reasoning and explanation offered by the authors. While studying the textbook, the teacher is likely to get many new ideas about how it can be taught and the teacher becomes better equipped for classroom teaching.

The textbook is also like a model that helps the teacher to create more material. She can think of ways in which she can improve and go beyond to what the textbook presents. In this chapter, we will discuss some ways of studying the textbook and take up exercises which are helpful in understanding the textbook more deeply. It is preferable that textbook study is taken up in a group in the school, or in teacher workshops. When questions and observations are shared and discussed, one's understanding becomes more thorough and deeper.

Fig. 12.3

Fig. 12.4

Fig. 12.5

We will now suggest some questions and issues that can be studied and discussed in a group. We will use the NCERT textbook, *Math-Magic* for Class II for the purpose of illustrating with examples.

Skimming the textbook: Skimming the textbook means going through or browsing the textbook without fully reading it. This is a first step and helps in getting a general idea and feel of the textbook. Several points can be looked for while skimming.

- How many units are there and how are they organised?
- How is the content of each unit organised?
    - What kinds of exercises are there?
    - Are there enough activities suggested in each unit?
    - Is there a general pattern in the units; are there titles or sections that appear repeatedly, like games or certain types of exercises?
    - How does the organisation of the textbook reflect the approach recommended in the curriculum framework?
- Are there notes or points for the teacher?
- Which topics are given more space?

While skimming the book, some doubts and queries are likely to arise. These doubts can be taken up while analysing the textbook in more detail. For example, if while skimming you come across a chapter titled **'Footprints'** then you might wonder *'what role do footprints play in mathematics?'* This question can be given more attention while reading the textbook more carefully.

Reading the preface: The 'preface' or the 'foreword' gives the reader an understanding of the vision and philosophy underlying the textbook and also usually gives the broad aims. There may be sentences or phrases that may not be fully clear in the first reading. Take, for example, this sentence from the foreword to the NCERT textbook, *'The textbook attempts to enhance this endeavour by giving high priority and space to opportunities for contemplation and wondering, discussion in small groups and activities requiring hands on experience'* (Math–Magic, NCERT 2007). What does this sentence convey? It suggests that the textbook tries to encourage thinking, discussions and activity based learning. The question that arises naturally is, how does the textbook try to achieve this? What is expected from the teacher in achieving these aims?

Skimming of each chapter: As you skim each chapter, the following points can be kept in mind. Does the title of the chapter give an idea of what it deals with? What are the major concepts dealt in the chapter? What are the headings, subheadings and sections in the chapter? What methods have been used to explain concepts and to promote students' reasoning? What assessment methods have been used? The *Math-Magic* textbook, for example, has adopted strategies like – stories, poems, narratives, activities, games, visuals, to enhance students' interest and understanding. Again, this might give rise to queries that can be taken up in detail later.

Looking at the visuals and graphics: In textbooks written for young children the visuals play a crucial role, and are often more important than the text since children are still developing their reading skills. As you read the textbook, you can focus on the visuals and ask what their purpose is. Authors and illustrators often think carefully before the illustration is finalised. Sometimes the illustration may support the text, and sometimes it replaces the text. Visuals can create the context in much more rich detail and more effectively than written text for young children. For example, in *Math-Magic* for Class II, there is an illustration of a garden near the pond where many migratory birds come and go, and the children are shown enjoying watching the birds. This context is created through an illustration without any text. As a teacher, we can think about how such visuals can be used to stimulate classroom discussion, and how the context depicted can be explored through questions.

Listing and classification: One can note down the questions that arise through a preliminary reading of the textbook and try to classify them. Some questions will be related to pedagogy and concepts, others will be related to the way the textbook and the chapters are structured. These questions can be taken up for discussion and for an analysis of the

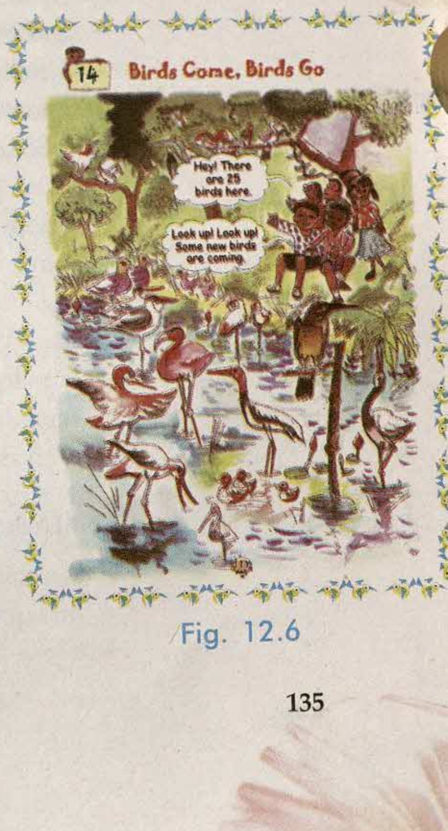

14
Birds Come, Birds Go
Hey! There are 25 birds here.
Look up! Look up! Some new birds are coming.

Fig. 12.6

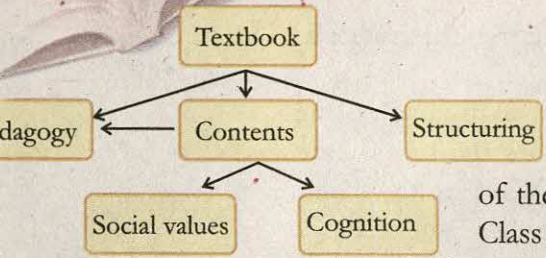


Fig. 12.7

textbook. Figure 12.7 shows what some of the broad categories of the questions may look like.

We will now take a closer look at one of the chapters in *Math-Magic,* the textbook for Class II, as an example that can be taken up for study and discussion.

Example for Analysis: Chapter Titled 'Jugs and Mugs' in *Math-Magic* for Class II

Some preliminary questions about the chapter that arise are:

- Why is the chapter called 'Jugs and Mugs'? As we examine the chapter, we find that it deals with the concept of the volume or amount of liquid in a container.
- Why is the chapter not titled 'volume of liquids'? Words such as 'volume' or 'capacity' are not yet part of the child's daily life experience. It is better to use familiar words in the title which will attract the child's attention.
- One can ask if the progression of ideas comes across clearly by looking at the headings and subheadings? In the case of this chapter, one does not grasp the progression of ideas clearly through the headings. One needs to look at the content of the chapter for this.
- Looking at the footnotes, we realise that some notes have been provided for the teacher. Although the notes describe the story and the activity to be done briefly, they do not explain what the objective of the lesson is or why the approach through the story has been taken. This is something that needs to be analysed and understood. However, the instructions for the activity have been explained clearly, including suggestions regarding local materials, and the kind of drink that can be used in place of those materials described in the textbook.

Analysis of the Concepts

- We find that the main concept in this chapter is the volume of liquids in containers. This concept is reinforced through activities that require judging or estimating how much liquid is in a container, and making comparisons between different liquid volumes.

- Does the textbook recognise that children already have a concept of liquid volume from their everyday experiences? How is this knowledge used to introduce the lesson?
- We see that a story about a thirsty crow has been included in the chapter. Why have the authors decided to use a story here? How is the story related to the main concept? Can children understand the story? Does it succeed in making them recall what they know from everyday experience? Does it succeed in making them focus on the relevant quantity, namely, the amount of water in the jug?
- How can the story be connected to the other activities like finding how much water can be filled in different containers? (The amount of liquid that can be filled in a container is called the capacity of the container.)
- One of the problems that children have with the concept of volume is understanding how to compare the volumes in different shaped containers. Young children often say that there is more water in a thin, tall glass than a wide glass because it looks tall and, hence seems to be more. (See chapter on 'Measurement') The question that one can ask is, does the textbook give enough experience to the children to explore the relation between shape and volume?
- Are the activities sufficiently interesting to the children? Can they be done in the classroom? What preparation is needed for the activities?
- Are the activities really necessary? Can the concepts be communicated only through pictures? (This is an important issue and needs careful discussion.)
- If we carefully go over the story of the thirsty crow, we see that it involves both the concepts of volume of a liquid and volume of a solid, Are the children of Class II ready to learn about the concept of volume of a solid? Why is this concept more difficult than volume of a liquid?
- If we look at the textbook for higher classes, we find that

the concept of volume of a solid is discussed in the Class IV NCERT textbook. Is it a good idea to revisit the story of the thirsty crow at that time?

- Does the crow's idea actually work? Can we check this by actually doing the experiment? Can the children do the experiment? Under what conditions does the water level fail to rise by putting stones in the jug? For example, what happens when there is only a little water in the jug?
- Is it possible to introduce group work based on the story?
- What are the points of assessment in relation to the concepts presented in this chapter?

OTHER ISSUES

- How can one go beyond the story and the activity? The teacher, for example, can ask children to find out other strategies for drinking the water in the jug, and discuss them in a larger group. (For example, one can use a straw.) The teacher can also tell the children the well-known story of 'The fox and the crane', in which the fox serves soup in a plate and the crane is unable to drink it. The teacher can ask the children to suggest some strategies that would allow the crane to drink the soup.
- Can the story be extended into a discussion about the importance of water, and of water scarcity?
- Is the chapter presented in a gender sensitive way? What support can it give to a teacher who is trying to make her class gender fair?

These questions are only suggestions to initiate a discussion about the textbook—its structure, content and objectives. A teacher will continue to study the textbook as she teaches through the term or the year. As the teacher develops a deeper understanding of the textbook, she not only becomes more expert at teaching, but to an extent, also gains freedom from the textbook to invent and explore alternative approaches.

13

Language and Mathematics Learning

Language makes it possible for us to communicate with one another, to share our thoughts and to generate new ideas through discussion and thinking together. Language plays a crucial role in the construction of knowledge. Hence we need to think carefully about the role language plays in learning mathematics.

Every discipline has a specialised language, and mathematics as a discipline also has its own language. It also borrows words from everyday language, but gives them special meanings. For example, the meaning of the word 'space' in common language is different from the meaning in mathematics. '*Jod*' in mathematics means addition, while in common language it has a broader meaning which includes the joint between two bones. Similarly *jama* means addition in mathematics, but in everyday language it could also mean 'freeze'. 'Division' has different meanings in everyday language, e.g., 'I am in III Class B division', 'cell division', etc., but in mathematics it has a special meaning. Thus familiar words are used with 'different' and 'unfamiliar' meanings in mathematics.

Fig. 13.1

Fig. 13.2

We commonly hear that the language of mathematics is precise. In mathematics we aim at precision so that we can make and prove general statements about the properties of mathematical objects. For example, read these two statements and think about what is different about them.

- Three intersecting lines make a triangle.
- When three lines intersect pairwise at three distinct points, they form a closed figure called a triangle.

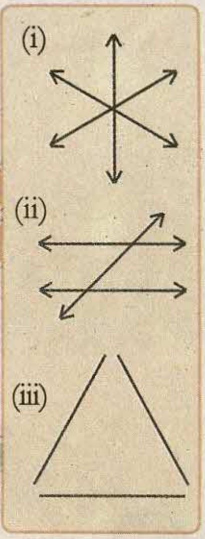


Fig. 13.3

Both these statements are about a mathematical object, namely, triangle. If we think about the first statement carefully, we see that it includes figures which are not triangles. For example, all the three lines may intersect at one point Fig. 13.3 (i), or two of the three lines may be parallel Fig. (ii). In both cases, no triangle is formed. Further, three line segments may form an open figure as in Fig. 13.3 (iii). The open figure may look like a triangle, but is not really a triangle because it is not closed. It does not have the universal property of triangles that the three angles of a triangle add to two right angles or 180 degrees. In fact, it does not even have three angles. The second statement is also incomplete, since only line segments form the triangle, the rest of the line (outside the triangle) is not a part of the figure. So we could make a more precise statement:

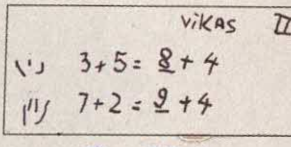
When three line segments meet at their endpoints so that they form a closed figure, the figure is called a triangle.

Mathematical definitions need to be precise because we use them to prove general facts (called 'theorems') in mathematics. We need to encourage children to make their mathematical language precise. A good way to do this is to encourage discussion about what they say – some child may interpret the statement in a different way, not in the way the child who made the statement intended. This can be a feedback for the child to change her statement so that it is clearer and more precise. Sometimes the statements have to be made longer and more elaborate to make them precise. This is alright. In mathematics, we often prefer precision combined with brevity or conciseness. But this is not always necessary in classroom discussion.

The use of symbols and signs is yet another part of mathematical language. Symbols and signs are a critical part of mathematics since they are a compact and efficient way of showing mathematical relationships. They can be manipulated to answer questions – when we do calculation in arithmetic or when we solve equations in algebra, we are manipulating symbols. In algebra, we also use symbols for other purposes: we may manipulate them to prove something, or to explain some pattern, or to gain mathematical insight and to discover new mathematics.

Think of how we can prove that the sum of two odd numbers is always even.

The language of symbols is new and unfamiliar to children and we need to take care that they understand them in the right way. Many children confuse the multiplication sign for the addition sign. This may be because of the unfamiliarity with the multiplication operation, or because they have not noticed the difference between the two signs. Look at the work done by Vikas of class 2. The '=' sign also has difficulties. Many children understand it as standing for the instruction, 'do something and write the answer'. This is the reason why, for the fill in the blank question: 3 + 5 = _ + 4, they write '8' in the blank instead of the correct answer '4'.



VIKAS II

(i) 3+5 = 8 + 4

(ii) 7+2 = 9 + 4

Fig. 13.4

Transition from common language to mathematical language

When children begin solving simple problems presented through words in the mathematics class, they begin working with mathematical language. 'How many?' 'How many altogether?' 'How many are left?'- all these are examples of the use of mathematical language. Children mix such mathematical language with their ordinary, everyday language while discussing a mathematical problem.

More complex mathematical language involves the use of mathematical vocabulary: addition, subtraction, tens, ones, odd numbers, even numbers, etc. But it is not essential to insist on using complex mathematical language. In fact, it is better not to insist on using formal mathematical terms till children feel the need for such language. When children are thinking, they may invent their own creative ways of using their existing vocabulary to convey mathematical ideas. A child in Class I in an English medium school said '7 is bigger than 5. 9 is bigger than 7 and 9 is more bigger than 5.' The child is discovering an important concept and mathematical relation here, but is using the limited vocabulary that is available to her. She is apparently pointing out that the difference between 9 and 5 is bigger than the difference between 7 and 5.

In making the transition to using mathematical language confidently, children require a lot of opportunities to use the language. If they are not encouraged to speak and discuss freely, then they will not learn how to use mathematical language. The teacher not only needs to encourage them to speak, but also to listen to them and support them so that their mathematical language capability develops.

CHILD'S LANGUAGE – A RESOURCE

When the child comes to school she brings many mathematical ideas to the classroom. These ideas are often expressed in a mathematical language which is a part of the culture and environment that the child is growing up in. A lot of mathematics comes from day-to-day experiences, from the street or market, from folklore or from anecdotes which are specific to the local environment of the child. If the child lives in a vegetable sellers' colony, then *ek pao, do pao, teen pao* will be familiar to the child. It is better to use these words to introduce the corresponding fractions rather then 'three fourths', 'one fourth', etc. The mathematical ideas that children bring to the class from their culture, and the language associated with such ideas is a rich resource for the teacher, something that the teacher can build on. Using such a resource, the teacher can make a connection with the more formal mathematical concepts and language, bringing them within the child's reach.

Children learn by constructing their knowledge in the classroom. A critical part of constructing knowledge is the sharing of ideas with peers in the classroom. Such sharing takes place in and through language. In the course of sharing, new thoughts and ideas are generated. While doing any activity, the child makes an attempt to understand her own or others' actions and this happens when children communicate with each other. This communication is possible only through a language that the children are comfortable in. For example, while exploring a

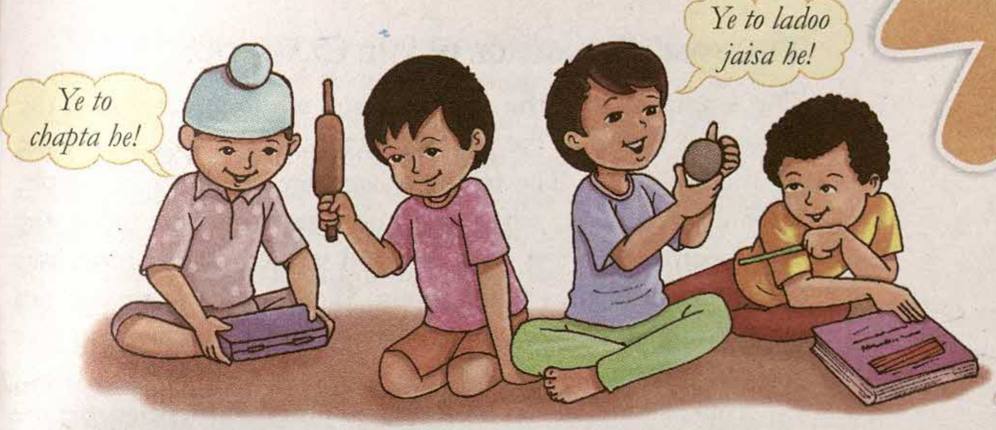

Ye to chapta he!
Ye to ladoo jaisa he!

Fig. 13.5

spherical object, the child may relate it with *ladoo* and calls it '*ladoo jaisa*', instead of '*golakaar*'. As a teacher should I accept and encourage this response? Notice that when the child uses the phrase, '*ladoo jaisa*', for a spherical object, she is making correct categories. She is making sense by connecting the properties of the spherical object with the properties of *ladoo*. So even though the child is not using the formal vocabulary, she is making conceptual progress and therefore we need to encourage her in using her own language.

In this way, the child can use the language she is comfortable with in describing things, noticing subtle differences. When young children explore shapes, they may use many words to make subtle distinctions between properties – '*yeh chapta hai, yeh lamba hai, yeh nokeela hai*', etc. By using such words, children learn from each other and their language competency develops. One may think that the development of children's language should happen in the language class, not in the mathematics class. But there is an important part of language that is informal, but still mathematical. This language can only develop in the mathematics class if children are given an opportunity. It is extremely important for the teacher to provide an opportunity in the mathematical classroom for the development of such informal language. Only then can the child learn more formal mathematical language. As teachers, if we allow and make use of the resource of child's own language, we are actually allowing child to think, understand and generate new ideas.

TEACHER'S LANGUAGE IN THE CLASSROOM

The language that the teacher uses while speaking in the classroom is one of the primary factors in shaping the culture of the classroom. The teacher's language needs to be simple, friendly and clear, when she gives explanations using the blackboard, when she gives oral and written instructions, asks questions or shapes a discussion. If as teachers, we are not careful, our speech can be ambiguous and vague and can give rise to misconceptions.

For instance, when we explain something written on the blackboard, what we are pointing to may not match with what we are saying. Sometimes what is said is vague. While explaining the algorithm of addition on blackboard for example, the child often hears vague phrases like – *inko jodo, ab jo number ayaa usko upar lelo, dahai ko upar likho, usko neeche utaro.* Such vague pointers may confuse the child.

One should also be careful about the oral instructions given in the class. To take a simple and familiar example, the teacher may say 'raise your right hand' and make a gesture with her right hand. Some children will raise the right hand and some will raise the left hand. The teacher needs to remember that her right hand is on the left side for the children sitting opposite to her. Thus while giving instructions, the teacher needs to take into account the child's perception.

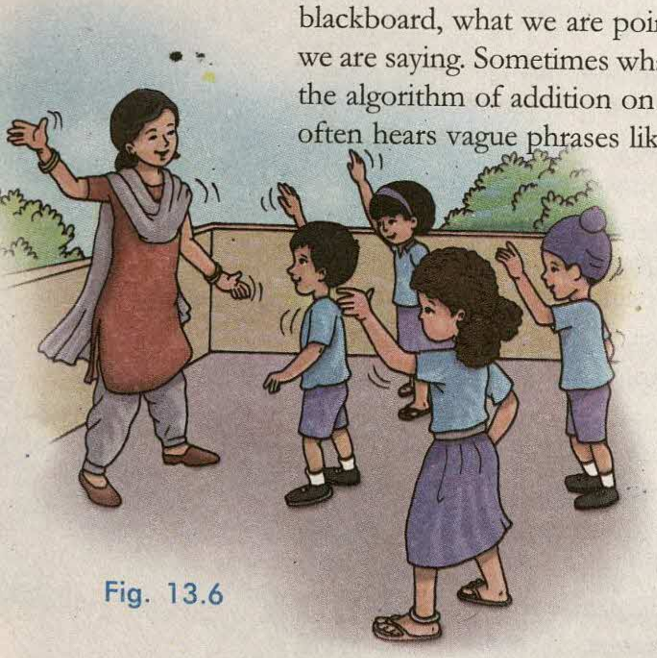


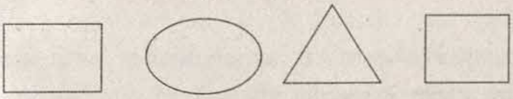
Fig. 13.6

Sometimes the sentences are also loaded with multiple instructions, which is difficult for the child to comprehend as well as to carry out. An instruction like 'colour the largest circular figure red and the smallest triangular figure blue' is complex both conceptually and in terms of language. We need to break instructions down to simple sentences that the child can follow. In this case, we could break the instruction into two simpler sentences.

The teachers' speech is often loaded with commands. Children hear many of them: '*sabhi prashno ke sahi sahi uttar do, ghatao, khali sthan bharo*'. Such questions can be asked in a more child friendly manner. Instead of '*khali sthan bharo*' one can ask an interesting question like, '*mohan ki billi ne kuch sankhyayain mita di hain, kya aap mohan ki madad karenge unhe dubra se likhne main?*'. Here we are asking the child to help Mohan, and this may make her more involved.

Often the questions asked are also close ended, which give little space to child to think and act on her own. Creativity, innovative ideas spring up only when enough space is given to the child's thoughts. Look at the two questions below and decide which one is more appropriate?

(a) Use the following shapes to make a Joker



(b) Using shapes like these make a picture of your choice.

In question (a) child has no space to think differently. Every child in the class will make a joker. Whereas, in question (b) which is asked in a slightly different manner, it opens the whole new world of shapes and figures which child can explore and make. When we give children an open choice, we convey that all their ideas are acceptable. Acceptance of child's idea is very crucial to allow the child to think creatively and innovatively.

LANGUAGE OF THE TEXTBOOK

Fig. 13.7

The language used in the textbooks is also important. Often the style of language in a textbook determines the style of language that will be used in the classroom. If the textbook language is very formal, the classroom language also tends to be formal and the interaction is restricted. If the textbook language is informal and understandable to a child, the classroom interaction may also be better. Sometimes the language of the textbook is too abstract for the child. For example, *10 rupaye ke note main kitne 2 rupaye ke note hain*? The child may wonder where the 2-rupee notes are in the 10-rupee note. This question can be asked in a better way like *dus ka note todvane par kitne do do ke note (ya sikke) milenge*?

Another questions from a textbook: A*arushi ki kaapi main 54 panne hain, woh 27 panno pe likh chuki hain, kitne panno par likhna shesh hai*? The child may think, what if Arushi does not write on the rest of the pages? The word '*shesh*' which means remaining confuses the child. This question could be made simpler and clearer by asking: *Aarushi ki kaapi main 54 panne hai, woh 27 panno pe likh chuki hain, kitne panne khali hai*?

Sometimes questions contradict the experience of children in real life. For example: *Wickets ke beech ki doori 22 kadam hain, vijay ko ek run poora karne ke liye kitne kadam daudna padhega*? The players don't run in steps, they just run as fast as possible. Thus if we are not careful about the language used, it causes confusion in the child's mind.

Textbook language is also often heavily loaded with formal terminology. For example, consider these two questions—

(a) *Niche de gayi sankhyaon ko aarohi evam avrohii kram main likho.*

(b) *Niche Samooh ke har khiladi ne kitne run banaye diye gaye hain, inhe badhte aur ghate kram main likhe.*

The objective of the first question is to test the child's understanding about ascending and descending order. The second question too has the same objective. But the second one is closer to the child's experience; its language is not loaded with technical words such as *aarohi kram/avrohi kram* (ascending and descending order). Many textbook writers attempt to make the text free from heavy terminology and keep the language close to the child's experience. Such texts help the child in comprehension which further aids problem solving skills.

MATHEMATICAL LANGUAGE AND WORD PROBLEMS

Most of our children face problems when they have to change a word problem to a mathematical statement. Not enough attention is given to this task in the classrooms. Consider this problem: Rohan has two pencils. Shikha has five pencils. How many more does Shikha have? Many children would respond to this problem by doing 2 + 5 = 7, which is incorrect. They may have been misled by the word 'more', which in English means 'greater amount or quantity' and may have decided to add. The child knows that addition gives a result, which is 'more' or greater in quantity or amount, than each of the starting quantities.

Children end up writing incorrect mathematical facts or carrying out the wrong operation, since they do not really comprehend the statement. They decide on the basis of 'word cues' (or word clues) that they pick up, that is, words that are frequently used and are familiar. Sometimes the language used in word problems is not simple and beyond the child's ability to comprehend.

Since word problems are an important part of school mathematics, we need to plan carefully so that students develop and strengthen their understanding of the mathematical language underlying word problems. Initially one can begin with picture stories so that visual clues help the children to understand the problem situation and the target quantity that they have been asked to find out. For example, Sama has 12 English story books and 16 Hindi story books.

How many books altogether?

Fig. 13.8

Rinku has 29 stamps.
Gurupreet gave him 5 more stamps.

Fig. 13.9

How many stamps altogether?

Another kind of task that is very useful is to ask the children to make a word problem in their own language. For example: if the word problem says – Ritu has 7 marbles and Rohit has 15 marbles. Who has more and by how many? The child could come up with something like *ritu ke pas saat kanche, rohit ke pas 15 kanche, rohit ke pass jyada kanche hai, ritu ke pas kum, rohit ke pass kitne jyada hai?* In this task when the child rephrases the word problem in her own language, teacher gets to know how much the child has comprehended. By doing this task, gradually children also become comfortable in understanding the mathematical language.

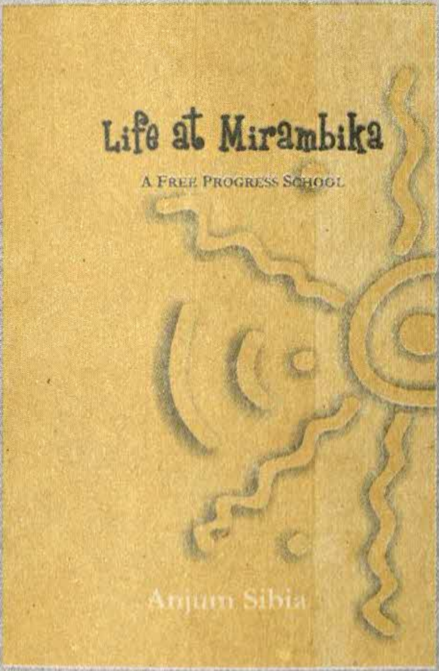
Another very important and helpful task, is to give a mathematical statement to the children and ask them to convert it into corresponding word problem. For example, make a word problem for the following:

(a) 7 – 3 = 4 (b) 6 + 4 = 10

Children come up with interesting word problems. One child when given such opportunity made an interesting word problem on multiplication, he said "*ek kamare main 12 kursiyan hain, batao kitne taange hai ?*" and this one response ignited the minds of the rest of the class and everyone made well articulated and meaningful word problems.

These are the few ways in which one can help children in doing word problems.

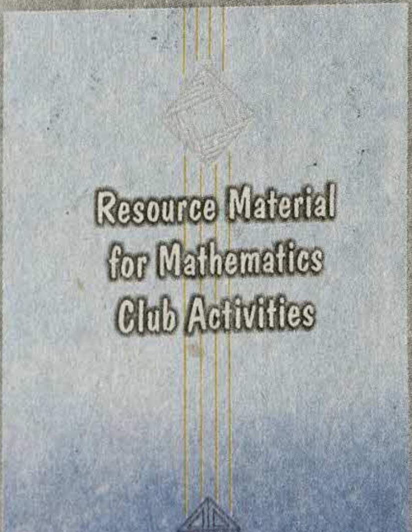
Learning mathematics in a meaningful manner will happen if we recognise that the language of mathematics is different from our day-to-day language. But it has its source in everyday language and must be connected to it. The extensive use of formal mathematical language in primary classes is actually a hindrance to child's expression and understanding. Rather, the teacher needs to pay attention to the process of using common language to express mathematical ideas, and to slowly develop and strengthen children's mathematical language. The language used in textbooks and that of teacher's language in the classroom is equally crucial. Simple sentences in child's own language will aid the comprehension which in turn will support problem solving skills. If we teachers make a conscious effort to analyse the language we use with our children and use our language wisely a big change can be seen in the teaching-learning process.



Life at Mirambika

A Free Progress School

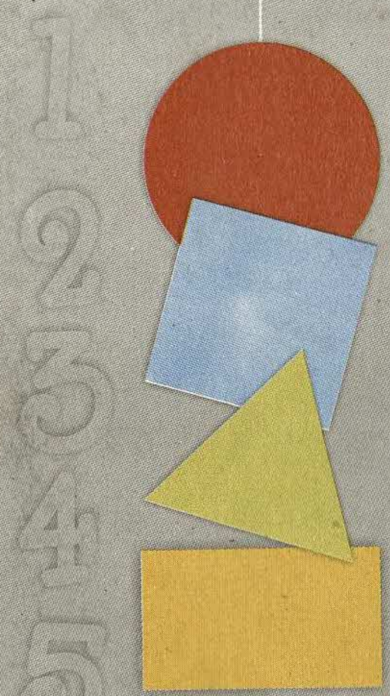
Rs. 90.00/Paperback/180 pp



Resource Material for Mathematics Club Activities

Rs. 80.00/Paperback/184 pp

For further enquiries, please visit www.ncert.nic.in or contact the Business Managers at the addresses of the regional centres given on the copyright page.



13057

विद्ययाऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

ISBN 978-93-5007-135-9